[71/A-12]

SEAT No.________

No of printed pages : 3

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - VI) EXAMINATION - 2019

Wednesday , 27^{th} March , 2019

MATHEMATICS : US06CMTH02

(COMPLEX ANALYSIS)

Time : 10:00 a.m. to 1:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ can be expressed as $f(z) =$........................

(a) $\bar{z}^2 + 2z$ (b) $\bar{z}^2 + iz$ (c) $\bar{z}^2 - 2iz$ (d) $\bar{z}^2 + 2iz$

(2) $\lim\limits_{z\to\infty} f(z) = w_0$ iff .... $= w_0$

(a) $\lim\limits_{z\to\infty} \frac{1}{f(z)}$ (b) $\lim\limits_{z\to 0} \frac{1}{f(z)}$ (c) $\lim\limits_{z\to 0} f\left(\frac{1}{z}\right)$ (d) $\lim\limits_{z\to 0} \frac{1}{z}$

(3) Singular point of $f(z) = \frac{z^3+4}{(z^2-3)(z^2+1)}$ are z = ...

(a) $\sqrt{3}$, i (b) $\pm\sqrt{3}$ (c) $\pm\sqrt{3}$, $\pm i$ (d) none of these

(4) $f(z) = \frac{z^3+i}{(z^2-3z+2)}$ is analytic in ....

(a) $\{\pm\sqrt{1}\ ,\ \pm 2\}$ (b) $\mathbb{C} - \{1\ ,\ 2\}$ (c) $\mathbb{C} - \{3\ ,\ \pm 2\}$ (d) $\{\ 1\ ,\ 2\ \}$

(5) $exp\left(\frac{2+\pi i}{4}\right) =$ ....

(a) $\sqrt{e/2}\,(1-i)$ (b) $\sqrt{e/2}\,(1+i)$ (c) $\frac{\sqrt{e}\,(1+i)}{2}$ (d) none of these

(6) $i \sin iy =$ ....

(a) $-\sinh y$ (b) $isinhy$ (c) $-isinhy$ (d) $cosiy$

(7) $e^{z_1} = e^{-z_2}$ then $z_1 =$ ....

(a) $-z_2 + 2n\pi i$ (b) $z_2 + 2n\pi$ (c) $-z_2$ (d) z_2

(8) Image of $y > 0$ under the transformation $w = (1+i)z$ is ....

(a) $u < v$ (b) $v < u$ (c) $0 < v$ (d) $v = u$

(9) Fixed point of $w = \frac{6z-9}{z}$ are ....

(a) 0 (b) i (c) 2 (d) 3

(10) Image of $y > 1$ under the transformation $w = (1-i)z$ is ....

(a) $u + v < 2$ (b) $v + u > 2$ (c) $u - v > 2$ (d) $u - v < 2$

Que.2 Answer the following (Any Ten) 20

(1) If $\lim\limits_{z\to z_0} f(z) = w_0$; $\lim\limits_{z\to z_0} g(z) = w_1$.Then prove that $\lim\limits_{z\to z_0} [f(z)g(z)] = w_0 w_1$.

(2) By using definition, prove that $\frac{d}{dz}(z^n) = nz^{n-1}$ for all $n \in \mathbb{N}$.

(3) Express $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ in terms of z and simplify the result, where $z = x + iy$.

(P.T.O)

(4) Prove that $f(z) = \bar{z}$ is nowhere analytic.

(5) Is $f(z) = 2xy + i(x^2 - y^2)$ entire function ? Verify it .

(6) Prove that $e^{-y}\sin x$ and $-e^{-y}\cos x$ are harmonic at each point of domain of xy-plane.

(7) Solve $e^z = w$ for z .

(8) Prove that $cosz_1 - cosz_2 = -2sin\left(\frac{z_1+z_2}{2}\right) sin\left(\frac{z_1-z_2}{2}\right)$.

(9) Find all zeros of $\cosh z$.

(10) Discuss the image of $w = (i+1)z + 2$.

(11) Find the image of line $x \geq c_1$, $c_1 > 0$ under the transformation $w = 1/z$ is the circle .Also.show the region graphically .

(12) Find the image of $a \leq x \leq b$, $c \leq y \leq d$ under the transformation $w = e^z$.

Que.3 (a) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point. Verify it. 6

(b) By using definition of limit prove that $\lim_{z \to z_0} (z^2 + c) = {z_0}^2 + c$, where c is complex constant. 4

OR

Que.3 (c) State and prove chain rule for differentiating composite functions. 6

(d) If $f(z) = \dfrac{x^3y(y-ix)}{z(x^6+y^2)}$, $z \neq 0$, $f(0) = 0$ .Prove that $f(z)$ differentiable at 0 . 4

Que.4 (a) Let $f(z) = u(x,y) + iv(x,y)$ be defined in some ϵ neighborhood of $z_0 = x_0 + iy_0$, suppose that the first order partial derivatives of u and v with respect to x and y exists every-where in that neighborhood ,if these partial derivatives all continuous at (x_0, y_0) and satisfies the C-R equations $u_x = v_y$, $u_y = -v_x$ at (x_0, y_0).Then prove that $f'(z_0)$ exists . 6

(b) Prove that $f(z) = (z^2 - 2)e^{-z}$ is entire function . 4

OR

Que.4 (c) Give a function which is differentiable at a point but not analytic at that point. Verify it. 3

(d) Find harmonic conjugate $v(x,y)$ of harmonic function $u(x,y) = \dfrac{y}{x^2+y^2}$. 3

(e) Give an example of function such that partial derivatives of its components satisfies the C-R equations at some points but function is not differentiable at that point. Verify it . 4

Que.5 (a) Prove that $\overline{exp(iz)} = exp(i\bar{z})$ iff $z = n\pi$, $n \in \mathbb{Z}$. 4

(b) Solve $e^z = -1 = \sqrt{3}i$. 3

(c) Prove that $\dfrac{d}{dx}(\sin^{-1} z) = \dfrac{1}{\sqrt{1-z^2}}$. 3

OR

Que.5 (d) Solve $\sin w = -i$. 4

(e) Evaluate $log(-1+\sqrt{3}i)$ and $\log(-1-\sqrt{3}i)$. 3

(f) Prove that $|sinhz|^2 = sinh^2x + sin^2y = cosh^2x - cos^2y$. 3

Que.6 (a) Find the image of $y > 1$ under the transformation $w = (1-i)z$ .Also sketch the region . 3

(b) Find the image of $0 < y < 1/2c$ under the transformation $w = 1/z$ .Also sketch the region . 3

(c) Find linear fractional transformation that maps the points $z_1 = -1$, $z_2 = 0$, $z_3 = 1$ *on to* $w_1 = -i$, $w_2 = 1$, $w_3 = i$ respectively. 4

OR

Que.6 (d) Prove that all linear fractional transformation that maps the upper half plane $Imz > 0$ on to the open disk $|w| < 1$ and the boundary Im z=0 on to the boundary of $|w| = 1$ is given by $w = e^{i\alpha}\left[\dfrac{z-z_0}{z-\overline{z_0}}\right]$, $Im\ z_0 > 0$. Also prove the converse . 6

(e) Prove that the transformation $w = \sin z$ is a one-one mapping of the semi infinite strip $y \geq 0$, $-\pi/2 \leq c_1 \leq \pi/2$ in the z-plane onto the upper half $v \geq 0$ of the w-plane. 4

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