[53/A-15] **SEAT No.__________** **No. of Printed Pages : 2**

SARDAR PATEL UNIVERSITY

Vallabh Vidyanagar

B.Sc.Sem-VI

MATHEMATICS: US06CMTH01

(Real Analysis-III)

25^{th} March'2019, Monday — 10.00 am to 01:00 pm

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) A bounded function f is said to be integrable over $[a, b]$, if there is fixed number I so that for every $\epsilon > 0$ there exists some $\delta > 0$ such that
(a)$|S(P, f) - I| < \epsilon$ for every partition P with $\mu(P) > \delta$ (b)$|S(P, f) - I| > \epsilon$ for every partition P with $\mu(P) > \delta$ (c)$|S(P, f) - I| < \epsilon$ for every partition P with $\mu(P) < \delta$ (d)$|S(P, f) - I| > \epsilon$ for every partition P with $\mu(P) > \delta$

(2) In usual notations, the Lagrange's form of remainder in Taylor's theorem is
(a)$\frac{h^{n-1}(1-\theta)^{n-p}}{p(n-1)!} f^n(a + \theta h)$ (b)$\frac{h^n(1-\theta)^{n-p}}{p(n-1)!} f^n(a + \theta h)$
(c)$\frac{h^n}{(n-1)!} f^n(a + \theta h)$ (d)$\frac{h^n}{n!} f^n(a + \theta h)$

(3) If μ is a mesh of the partition $P = \{x_0, x_1, x_2, \ldots, x_n\}$ for $[a, b]$ then for every $i = 1, 2, \ldots, n$
(a) $\Delta x_i = \mu$ (b) $\Delta x_i < \mu$ (c) $\Delta x_i \geq \mu$ (d) $\Delta x_i \leq \mu$

(4) A function f cannot be integrable over $[a, b]$, if it is
(a) Increasing over $[a, b]$ (b) Decreasing over $[a, b]$ (c) Continuous over $[a, b]$
(d) None of these

(5) If P is a partition of $[a, b]$, then
(a) $a \in P$, but $b \notin P$ (b) $a \notin P$, but $b \in P$ (c) $a \in P$ and $b \in P$ (d) $a \notin P$ and $b \notin P$

(6) If a function f has a finite number of points of discontinuity over $[a, b]$ then it is
(a) ..ot integrable over $[a, b]$ (b) integrable over $[a, b]$
(c) monotonic over $[a, b]$ (d) none

(7) A function $f(x)$ has a minimum at c if while x passes through c, f changes from
(a) an increasing to a decreasing function (b) a decreasing to an increasing function
(c) an increasing to a constant function (d) none

(8) For a bounded function f defined on $[a, b]$ and two partitions P_1, P_2 and $P^* = P_1 \cup P_2$
(a) $U(P_1, f) \leq U(P^*, f)$ (b) $U(P_1, f) < U(P^*, f)$ (c) $U(P_1, f) \geq U(P^*, f)$
(d) $U(P_1, f) > U(P^*, f)$

(9) A twice differentiable function $f(x)$ has a maximum at c if
(a)$f(c) = 0, f'(c) > 0$ (b)$f'(c) = 0, f''(c) > 0$
(c)$f'(c) = 0, f''(c) < 0$ (d)$f(c) = 0, f''(c) < 0$

(10) A refinement of a partition P contains
(a) At least one element less than the elements of P (b) At least one element more than the elements of P (c) All the elements are different from P (d) No element different from P

Q.2 Attempt any **Ten**. [20]

(1) Show that if two functions have equal derivatives at all points of $[a, b]$, then they differ only by a constant.

(2) Define: Stationary Point and Stationary Value.

(3) Show that $\sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$.

(4) A function f is integrable over $[a, c]$ and $[c, b]$. If $\int_a^c f dx = k$, $\int_c^b f dx = 3k$ and $\int_a^b f dx = 36$, then find k.

(5) Let $f : [0, 1] \to \Re$ be defined by $f(x) = 2.7$. Find $\int_0^1 f dx$.

(6) Prove that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$, where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(7) For a bounded function $f(x) = x^2, x \in [1, 7]$ and partition $P = \{1, 2, 5, 7\}$, find $U(P, f)$.

(8) State First mean value theorem of differential Calculus. [PTO]

(9) State Taylor's theorem.

(10) State the Second Fundamental theorem of integral calculus.

(11) In usual notations, prove that $m(b-a) \leq \int_a^b f dx \leq M(b-a), a \leq b$.

(12) Prove that the function $[x]$ where $[x]$ denotes the greatest integer not greater than x, is integrable on $[0, 4]$ and $\int_0^4 [x]dx = 4$.

Q.3

(a) State and prove Cauchy's mean value theorem. [5]

(b) A twice differentiable function f is such that $f(a) = f(b) = 0$ for $a < c < b$. Prove that there is at least one value ξ between a and b for which $f''(\xi) < 0$. [5]

OR

Q.3

(c) Prove Taylors theorem with Cauchys form of remainder. [5]

(d) Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ on $[2, 5]$. [5]

Q.4

(a) Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. [6]

(b) Show that maximum value of the function $(x-1)(x-2)(x-3)$ is $\frac{2\sqrt{3}}{9}$ at $2 - \frac{1}{\sqrt{3}}$. [4]

OR

Q.4

(c) If c is an interior point of the domain $[a, b]$ of a function f and is such that $(i) f'(c) = f''(c) = \cdots = f^{n-1}(c) = 0$ and $(ii) f^n(c)$ exist and is zero, then show that for n odd, $f(c)$ is not an extreme value, while for n even $f(c)$ is maximum or minimum Value according as $f^n(c)$ is negative or positive. [6]

(d) Examine the function $(x-3)^5(x+1)^4$, for extreme values. [4]

Q.5

(a) State and Prove Darboux's theorem for integration. [5]

(b) By using definition of integration, prove that the function $(3x+1)$ is integrable on $[1, 2]$ and find its integral value. [5]

OR

Q.5

(c) Show that a necessary and sufficient condition for the integrability of a function f is that every $\epsilon > 0$, there exists $\delta > 0$ such that for every partition P of $[a, b]$ with mesh less than δ, $U(P, f) - L(P, f) < \epsilon$. [5]

(d) Show that the product of two bounded and integrable functions on $[a, b]$ is also integrable. [5]

Q.6

(a) If a function f is bounded and integrable on $[a, b]$, then show that the function F defined as $F(x) = \int_a^x f(t)dt, a \leq x \leq b$ is continuous on $[a, b]$. Also, if f is continuous at a point c of $[a, b]$, then prove that F is derivable at c and $F'(c) = f(c)$. [5]

(b) If f is continuous on $[a, b]$, then there exists a number ξ in $[a, b]$ such that $\int_a^b f dx = f(\xi)(b-a)$. [5]

OR

Q.6

(c) Show that a function f is integrable on $[a, b]$ iff for $\epsilon > 0$, there exists $\delta > 0$ such that if P, P' are any two partitions of $[a, b]$ with mesh less that δ, then prove that $|S(P, f) - S(P', f)| < \epsilon$. [5]

(d) Prove that a bounded function f having a finite number of points of discontinuity on $[a, b]$ is integrable on $[a, b]$. [5]

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