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SARDAR PATEL UNIVERSITY
B.Sc. (VI Semester) Examination
Saturday, 13th April 2013
3 - 6 pm
US06CMTH05 – Mathematics/Graph Theory

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Choose the most appropriate option for the following and write it down in the answer-book. [10]

(1) Degree of pendant vertex is _________.
(a) 3 (b) 2 (c) 1 (d) 0

(2) An alternative sequence of vertices and edges in which no edge is covered more than once is called _________.
(a) walk (b) circuit (c) self loop (d) path

(3) In a connected graph there is a path between _________ pair of vertices.
(a) at least one (b) every (c) no (d) None

(4) A tree with n vertices has _________ edges.
(a) n (b) n+1 (c) n+2 (d) n-1

(5) A vertex with minimum ecentricity is called _________.
(a) diameter (b) centre (c) radius (d) none

(6) A spanning tree T of graph contains all the _________ of G.
(a) vertices (b) edges (c) regions (d) None

(7) By removing cut-set from the given graph, it becomes _________ graph.
(a) null (b) connected (c) disconnected (d) None

(8) Every connected graph has _________ spanning tree.
(a) at most one (b) at most two
(c) exactly one (b) at least one

(9) In a graph having 5 vertices and 4 regions, number of edges equal to ________.
(a) 3 (b) 5 (c) 7 (d) 9

(10) $K_{3,3}$ is _________ graph.
(a) planar (b) non-planar (c) disconnected (d) None

Q.2 Answer the following in short. **(Attempt Any Ten)** [20]

1. Define: Isomorphic graphs.
2. Describe utilities problem.
3. Define: Parallel edges with illustration.
4. What is Euler graph?
5. Explain the operation ring sum of two graphs.
6. Define: Arbitrary traceable graph with an example.
7. Define : Spanning tree with illustration.
8. Explain about branch of a spanning tree.
9. Define : Fundamental Circuit.
10. Define : Homeomorphic graphs with example.
11. Draw Kuratowski's first graph.
12. By using Euler's theorem prove that Kuratowski's first graph is non-planar.

Q.3

(a) Show that a simple graph G with n-vertices and k-components must have atmost $\frac{(n-k)\,(n-k+1)}{2}$ edges. [05]

(b) Prove that a graph G is disconnected iff its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in V_2. [05]

OR

Q.3

(a) If a graph has two vertices of odd degree, then show that there must be a path between them. [05]

(b) What is Königsberg bridge problem ? Solve it by using graph theory. [05]

Q.4

(a) Show that every tree has either one or two centre. [05]

(b) Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree. [05]

OR

Q.4

(a) Prove that a tree with n-vertices has n-1 edges. [05]

(b) Show that a connected graph G is an Euler graph iff it can be decomposed into circuits. [05]

Q.5

(a) Show that in a connected graph G any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set. [05]

(b) Discuss method of finding all spanning trees of a graph. [05]

OR

Q.5

(a) Prove that every circuit has even number of edges in common with cut-set. [05]

(b) Show that the minimum vertex connectivity one can achieve with a graph G of an vertices and e edges $(e \geq n-1)$ is $\left[\frac{2e}{n}\right]$. [05]

Q.6 State and prove Euler theorem. [10]

OR

Q.6 State and prove the necessary and sufficient condition for two planar graphs to be dual of each other. [10]

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