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SEAT No.________

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. Examinations: 2017-18 – VI SEM

Subject : Mathematics US06CMTH05 Max. Marks : 70

Graph Theory

Date: 04/04/2018, Wednesday Timing: 10:00 am - 01:00 pm

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] If degree of a vertex is zero then it is called
[A] a pendent vertex [B] an isolated vertex [C] circuit [D] path

[2] If there are 5 edges in a simple graph then the total of degrees of vertices of the graph is
[A] 5 [B] 10 [C] 15 [D] 20

[3] If terminal vertices of a walk in a graph are same then it is called
[A] an open walk [B] a closed walk [C] path [D] none

[4] A connecetd graph is an Euler graph if all vertices of the graph are
[A] of odd degree [B] of even degree [C] isolated [D] pendent

[5] An operation of edge deletion on a graph removes corresponding
[A] edge only [B] vertices [C] vertices and edges both [D] none

[6] The ring sum of two graphs does not include
[A] common edges [B] common vertices [C] pendent vertices [D] none

[7] Rank of a graph with 4 vertices, 6 edges and 2 components is
[A] 1 [B] 2 [C] 3 [D] 4

[8] A tree is a ____-connected graph.
[A] 0 [B] 1 [C] 2 [D] 3

[9] If graphs G_1 and G_2 are isomorphic and nullity of G_1 is 7 then nullity of G_2 is
[A] 7 [B] 14 [C] 21 [D] 49

[10] A planar graph with 7 vertices and 10 edges has ____ faces
[A] 5 [B] 7 [C] 9 [D] 10

Q: 2. Answer any TEN of the following. **20**

[1] Discuss Konigsberg bridge problem

[2] Define : (i) Subgraph (ii) Closed walk

[3] Define : (i) Edge disjoint subgraphs (ii) Length of path

[4] Draw all labeled trees with four vertices.

[P.T.O.]

[5] Define Complete graph with an example.

[6] Explain Fusion of vertices with an example.

[7] Explain Spanning Tree with an example.

[8] Describe network flows

[9] Prove that the edge connectivity of a graph G can not exceed the degree of a vertex with the smallest degree in G.

[10] Discuss Kurtowski's First graph.

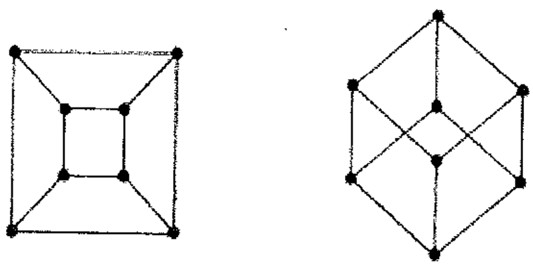
[11] For a simple connected planar graph with n-vertices , e-edges ($e > 2$) and f-regions prove the following.

(i) $e \geqslant \frac{3}{2}f$ (ii) $e \leqslant 3n - 6$

[12] Define Circuit correspondence

Q: 3 [A] Prove that a graph G is disconnected *iff* its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and other in subset V_2 5

[B] Explain Isomorphism between two graphs. and examine whether following pair of graphs is isomorphic or not.



5

OR

Q: 3 [A] Prove that a simple graph with n vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 5

[B] If a graph (connected or disconnected) has exactly two vertices of odd degree then prove that there must be a path joining these two vertices. 5

Q: 4 [A] Prove that a tree with n-vertices has $n-1$ edges. 5

[B] Prove that in a complete graph with n vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits, if n is an odd number $\geqslant 3$. 5

OR

Q: 4 [A] Prove that every tree has either one or two centers. **5**

[B] Prove that a connected graph G is an Euler graph *iff* all vertices of G are of even degree. **5**

Q: 5 [A] Describe a method to find all spanning tree of a graph. **5**

[B] Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G. **5**

OR

Q: 5 [A] Prove that every circuit has an even number of edges in common with any cut-set. **5**

[B] Prove that every cut-set in a connected graph G must contain atleast one branch of every spanning tree. **5**

Q: 6 [A] Prove that 2-isomorphic graphs have circuit correspondence. **5**

[B] State and prove Euler's theorem for planar graphs. **5**

OR

Q: 6. Define Planar graph and prove that a graph has a dual *iff* it is planar. **10**

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