[5IA-20] **SEAT No.________** No of printed pages : 3

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - VI) EXAMINATION - 2018

Monday , 2^{nd} April , 2018

MATHEMATICS : US06CMTH04

(ABSTRACT ALGEBRA - 2)

Time : 10:00 a.m. to 1:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) .... is a ring with zero divisor but not an integral domain .

(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) none of these

(2) .... is regular element of $\mathbb{Z}_9$.

(a) 3 (b) 4 (c) 6 (d) none of these

(3) In ring $R = \{a + b\sqrt{-5} \ / \ a , b \in \mathbb{Q} \}$, $(-1+2\sqrt{-5})^{-1} =$ ....

(a) $\frac{1-2\sqrt{-5}}{21}$ (b) $\frac{-1-2\sqrt{-5}}{5}$ (c) $\frac{-1-2\sqrt{-5}}{21}$ (d) $\frac{-1+2\sqrt{-5}}{21}$

(4) Quotient field of ring of Gaussian integer is ....

(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $\mathbb{Z}+i\,\mathbb{Z}$ (d) $\mathbb{Q}+i\,\mathbb{Q}$

(5) $Z/5Z =$ ....

(a) Z_5 (b) Z (c) Z_4 (d) $1/5$

(6) If I is ideal in ring R and $a + I = I$ then ....

(a) $a = 0$ (b) $a = I$ (c) $a \in I$ (d) none of these

(7) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, $1 + 2\sqrt{-5}$ is .... in R .

(a) not unit (b) not irreducible (c) prime (d) unit

(8) In $\mathbb{Z} + i\mathbb{Z}$, gcd of 2 and $-1 + 5i$ is ....

(a) $2+i$ (b) $2-i$ (c) i (d) $1-i$

(9) If R is field , $f(x), g(x) \in R[x]$ then $deg\ (fg)$..........$deg\ f + deg\ g$.

(a) $>$ (b) $\leq$ (c) $=$ (d) $\geq$.

(10) Let R be Euclidean domain , $a, b \in R$, a is proper divisor of b then $d(b)$..................$d(a)$.

(a) $=$ (b) $\leq$ (c) $>$ (d) $<$

Que.2 Answer the following (Any Ten) 20

(1) Prove that ring C[0,1] is ring with zero divisor under pointwise multiplication.

(2) Prove that $f(0) = 0$,where $f : R \to R'$ be ring homomorphism .

(3) Prove that $f : \mathbb{Z} \to \mathbb{Z}_n$ defined by $f(i) = \bar{i}$ is ring homomorphism .Is it one-one.?

(4) Let $R = C[0, 1]$.Prove that $I = \{ x \ / x \in R,\ x(1/2) = 0\}$ is an ideal in R.

(P.T.O.)

(5) Find Z_6/I , where $I = \{\bar{0}, \bar{2}, \bar{4}\}$.

(6) Let I be any ideal in ring R and $p : R \to R/I$ be the projection map.If $J \supset I$ is ideal in R then prove that $p(J) = J/I$ is ideal in R/I .

(7) Prove that $4 + 3\sqrt{-5}$ is irreducible in the ring $\{a + b\sqrt{-5}/a, b \in Z\}$.

(8) Prove that every Euclidean domain has unit element .

(9) If R is ring with unit element 1 then prove that $Ra \subset Rb \Leftrightarrow b/a$.

(10) Prove that any two elements of unique factorization domain have a gcd.

(11) Let F be a field and $f(x) \in F[x]$ be a polynomial of degree n .Then prove that $f(x)$ has at most n distinct roots in F.

(12) Find content of f ,where $f(x) = 2x^2 + (1+i)x - 2 \in R[x]$, where $R = Z + iZ$.

Que.3 (a) Prove that Z_p is a field iff p is prime . 4

(b) Prove that the characteristic of every field is either 0 or prime . 3

(c) Let $f : R \to R'$ be a ring homomorphism ,then prove that $Ker f$ is subring of R and $f(R)$ is subring of R'. 3

OR

Que.3 (d) Prove that $R = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} / \ a, b, \in \mathbb{R} \right\}$ is a non-commutative ring without unit element. 5

(e) State and prove Cayley's theorem for rings . 5

Que.4 (a) Prove that every field is a simple ring . State and prove under which condition converse hold . 5

(b) Let R be any commutative ring with unit element 1 , I be any ideal in R then prove that $(R/I, +, .)$ is a commutative ring with unit element $1 + I$. 5

OR

Que.4 (c) State and prove First isomorphism theorem for ring. 5

(d) Prove that every maximal ideal in a commutative ring with 1 , is a prime ideal . Under which condition converse hold ? Verify it. 5

Que.5 (a) Let$R = \{a + b\sqrt{-5}/a, b \in Z\}$. Show that $\alpha\gamma$ and$\beta\gamma$ have no gcd in R , where $\alpha = 3$; $\beta = 1 + 2\sqrt{-5}$; $\gamma = 7(1 + 2\sqrt{-5})$. 5

(b) Prove that every principal ideal domain is factorization domain. 5

OR

Que.5 (c) Prove that every prime element is irreducible in integral domain with unit element 1 . Does the converse hold ? Verify it. 6

(d) Prove that any two elements of principal ideal domain have a gcd. 2

(e) Find gcd of $2 + 3i$ and $4 + 7i$ in ring $\mathbb{Z} + i\mathbb{Z}$. 2

Que.6 (a) State and prove Gauss theorem. 5

(b) Let R be a unique factorization domain .Then prove that the product of two primitive polynomials over R is also a primitive polynomial. 5

OR

Que.6 (c) If F is field then prove that F[x] is a unique factorization domain. 5

(d) If every irreducible element is prime in factorization domain R ,then prove that R is a unique factorization domain. 5

(3)