Sc

(57|A-16) SEAT No.________ **No. of Printed Pages : 3**

Sardar Patel University, Vallabh Vidyanagar

B.Sc. Examinations: 2017-18 (VI SEM)

Subject : Mathematics US06CMTH03 Max. Marks : 70

Topology

Date: 31/03/2018, Saturday Timing: 10:00 am - 01:00 pm

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] The discrete topology on a non-empty set X is ...... its indiscrete topology
[A] coarser than [B] finer than [C] not comparable with [D] none

[2] In $(\mathcal{R},\mathcal{U})$, the set $[0,1]$ is
[A] Open [B] Closed [C] Open as well closed [D] Neithr open nor closed

[3] The topologies $T_1 = \{\emptyset, \{a\}, X\}$ and $T_2 = \{\emptyset, \{a\}, \{a,b\}, X\}$ for $X = \{a,b,c\}$ are such that
[A] T_1 is finer than T_2 [B] T_2 is finer than T_1
[C] T_2 is coarser than T_1 [D] they are non-comparabale

[4] If A is a closed set in a topological space then
[A] $A \subset A'$ [B] $A^- \neq A$ [C] $A = A'$ [D] $A' \subset A$

[5] The set of all cluster points of empty set $\emptyset$ in $(\mathcal{R},\mathcal{U})$ is
[A] $\emptyset$ [B] $\mathcal{R}$ [C] $\mathcal{Q}$ [D] none

[6] In $(\mathcal{R},\mathcal{U})$ which of the following is not dense
[A] $\mathcal{R}$ [B] $\mathcal{Q}$ [C] $\mathcal{J}^+$ [D] $\mathcal{R} - \mathcal{Q}$

[7] Every non-empty and bounded above subset of R posseses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

[8] If $I_1 = (0,1)$ and $I_2 = (1,2)$ then the subspaces $(I_1, \mathcal{U}_{I_1})$ and $(I_2, \mathcal{U}_{I_2})$ of $(R,\mathcal{U})$
[A] both are compact [B] are homeomorphic
[C] both are disconnected [D] none

[9] In a T_1 space the complement of every singleton set is
[A] closed [B] open [C] closed and open both [D] neither open not closed

[10] For a T_1 space (X,T) if $a,b \in X$ then $X - \{a,b\}$ is
[A] closed [B] open [C] closed and open both [D] neither open not closed

Q: 2. Answer any TEN of the following. 20

[1] Show that the sets $\mathbb{R}$ and $\emptyset$ are $\mathcal{U}$-open

[2] Define : (i) Indiscrete Topology (ii) Closed Set

(P.T.O.)

[3] For a set $X = \{1,2,3\}$ give a closed subset of X relative to the topology $\{\emptyset, X, \{1\}, \{2,3\}\}$. Is that open also?

[4] Let $(X, \mathcal{T})$ be a topological space. Find the set of all the cluster points of the empty subset of X

[5] Let $(X, \mathcal{T})$ be a topological space. Prove that if F is $\mathcal{T}$-closed subset of X and $p \in (X \sim F)$ then there is a $\mathcal{T}$-neighbourhood N of p such that $N \cap F = \emptyset$

[6] Define : (i) Interior of a set (ii) Bicontinuous function

[7] For $X = \{a,b,c\}$ consider the topology $\mathcal{T} = \{X, \emptyset, \{a,b\}, \{c\}\}$. Is (X, T) connected?

[8] Prove that indiscrete space is connected

[9] State the Least Upper Bound property of R

[10] Is every discrete space a T_1 space also? Why?

[11] Prove that every Hausdorff space is T_1-space

[12] Prove that the space $(R, \mathcal{U})$ is a T_2-space.

Q: 3 [A] Show that discrete topology satisfies all the conditions for becoming a topological space **5**

[B] Let $\mathcal{G}$ be a family of subsets of $\mathbb{R}$ as described below
(i) $\emptyset \in \mathcal{G}$
(ii) If $G \in \mathbb{R}$ and $G \neq \emptyset$ then $G \in \mathcal{G}$ if for each $p \in G$ there is a set $H = \{x \in \mathbb{R} / a \leqslant x < b\}$ for some $a < b$ such that $p \in H \subset G$.
Prove that $\mathcal{G}$ is an unusual nontrivial topology of $\mathbb{R}$ **5**

OR

Q: 3 [A] If $X = \{a,b,c\}$ then find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ for X such that $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 \subsetneqq \mathcal{T}_3$. Also find three more topologies for X which are non-comparable with each other. **5**

[B] Prove that finite union and arbitrary intersection of closed sets in a topological space are closed. **5**

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and A be a subset of X. Prove that $A \cup A'$ is $\mathcal{T}$-closed **5**

[B] Let $(X, \mathcal{T})$ be a topological space and $A \subset X$. Prove the following
(i) A is $\mathcal{T}$-open iff $Int(A) = A$ (ii) $Int(A)$ is the largest open subset of A **5**

OR

(P. T. O.)

Q: 4. If $(X, \mathcal{T})$ and (Y, Ψ) are topological spaces and f is a mapping from X into Y then prove that the following statements are equivalent 10

(a) The mapping f is continuous

(b) The inverse image of f of every Ψ-closed set is $\mathcal{T}$-closed set

(c) If $x \in X$ then inverse image of every Ψ-neighbourhood of $f(x)$ is a $\mathcal{T}$-neighbourhood of x

(d) If $x \in X$ and N is a Ψ-neighbourhood of (x), then there is a $\mathcal{T}$-neighbourhood M of x such that $f(M) \subset N$

(e) If $A \subset X$, then $f(A^-) \subset f(A)^-$

Q: 5 [A] If $(Y, \mathcal{T}_Y)$ is a compact subspace of a Hausdorff space $(X, \mathcal{T})$, then prove that Y is $\mathcal{T}$ closed. 5

[B] Let $(X, \mathcal{T})$ be a topological space and let Y be a subset of X. Prove that if the subsapce $(Y, \mathcal{T}_Y)$ is connected then so is the subspace $(Y^-, \mathcal{T}_{Y^-})$. 5

OR

Q: 5. Define a connected space and prove that the space $(R, \mathcal{U})$ is connected. 10

Q: 6 [A] If Y is a bounded and $\mathcal{U}$-closed subset of R, then prove that $(Y, \mathcal{U}_Y)$ is compact. 5

[B] If $(X, \mathcal{T})$ is compact and A is an infinite subset of X, then prove that A has atleast one cluster point in X. 5

OR

Q: 6 [A] If $(X, \mathcal{T})$ is a compact space, and if f is a $\mathcal{T} - \psi$ continuous mapping of X into R, then prove that f is bounded. 5

[B] Prove that every compact Hausdorff space is a T_3-space. 5