SC

SEAT No.________

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[51/A18]

SARDAR PATEL UNIVERSITY
B.Sc. (*SEMESTER – VI*) EXAMINATION-2018
March 28, 2018, Wednesday
10:00 a.m. to 1.00 p.m.
US06CMTH02(Complex Analysis)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Domain of $f(z) = \dfrac{z}{z^4 - 1}$ is ......
(a) $\mathbb{C} - \{\pm 1\}$ (b) $\mathbb{C} - \{0, \pm i, \pm 1\}$ (c) $\mathbb{C} - \{\pm 1, \pm i\}$ (d) $\mathbb{C} - \{\pm i\}$

(2) Cartesian form of $f(z) = z^2 - 5i\bar{z}$ is $f(z) =$ ......
(a) $f(z) = (x^2 - y^2 - 5x) + i(2xy + 5y)$ (b) $f(z) = (x^2 - y^2 - 5y) + i(2xy - 5x)$
(c) $f(z) = (x^2 - y^2 + 5x) + i(2xy - 5y)$ (d) $f(z) = (x^2 - y^2 + 5y) + i(2xy + 5x)$

(3) $\lim_{z \to z_0} f(z) = \infty$ iff .... $= 0$
(a) $\lim_{z \to 0} \dfrac{1}{f(z)}$ (b) $\lim_{z \to z_0} \dfrac{1}{f(z)}$ (c) $\lim_{z \to \infty} \dfrac{1}{f(z)}$ (d) $\lim_{z \to z_0} f\left(\dfrac{1}{z}\right)$

(4) If $f(z) = 2x^2 + i4xy$ then f is differentiable at ......
(a) 0 (b) $\{z \in \mathbb{C} / Im(z) = 0\}$ (c) $\{z \in \mathbb{C} / Re(z) = 0\}$ (d) none of these

(5) If $u(x, y) = 2x - x^3 + 3xy^2$ then......
(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} + u_{yy} = 1$ (c) $u_{xx} + u_{xy} = 0$ (d) none of these.

(6) $f(z) = \dfrac{z^3 + 4}{(z^2 - 3)(z^2 + 1)}$ is analytic in ....
(a) $\{\pm\sqrt{3}\ ,\ \pm i\}$ (b) $\mathbb{C} - \{\sqrt{3}\ ,\ i\}$ (c) $\mathbb{C} - \{\sqrt{3}\ ,\ \pm i\}$ (d) none of these

(7) $\sinh(2\pi i) =$ ....
(a) 0 (b) 1 (c) i (d) -1

(8) $\exp z$...............0 , $\forall\ z \in \mathbb{C}$.
(a) $\leq$ (b) $\geq$ (c) $=$ (d) $\neq$

(9) Image of a horizontal strip $3 < x < 7$ under the transformation $w = iz$ is ....
(a) $3 < u < 7$ (b) $3 < v < 7$ (c) $v < u$ (d) $u < v$

(10) Fixed point of $w = \dfrac{z-1}{z+1}$ are ....
(a) i (b) -1 (c) $\pm i$ (d) ± 1

Q.2 Attempt any Ten: [20]

(1) Prove that limit of function is unique, if it exist.

(2) Represent the region $3 < |z - 5i| < 7$ graphically in complex plane.

(3) Using definition of limit, show that $\lim_{z \to z_0} c = c$, where c is complex constant.

(4) Define: Analytic function, Entire function.

(5) Show that $u(x, y) = 3x^2y - y^3$ is harmonic in some domain of complex plane.

(6) Show that $f(z) = e^{ix+y}$ is nowhere analytic.

(7) Find all values of z such that $e^z = -1 + i\sqrt{3}$.

(8) Prove that $\cos^2 z + \sin^2 z = 1$.

(9) Evaluate $\log(e^4 i)$.

(P.T.O.)

(10) Find fixed points of the transformation $w = \dfrac{z-4}{z-3}$.

(11) Define Bilinear transformation. Is $T(z) = \frac{4z-6}{6z+9}$ a Bilinear transformation?

(12) Find the image of $1 < y < 4$ under the transformation $w = -5z$. Also sketch the region.

Q.3

(a) If f and g are differentiable then prove that fg is differentiable and $(fg)'(z) = f(z)g'(z) + f'(z)g(z)$. [05]

(b) By using definition of limit prove that $\lim_{z\to 2i} (2x + iy^2) = 4i$. [05]

OR

Q.3

(c) State and prove chain rule for differentiation of composite functions. [06]

(d) Prove that every differentiable function is continuous. Does the converse hold? Verify it. [04]

Q.4

(a) Prove that $f'(z)$ and $f''(z)$ exist everywhere and find $f''(z)$ for $f(z) = e^{-z}$. [05]

(b) Let $f(z) = u(x,y) + iv(x,y)$ and $f'(z)$ exist at $z_0 = x_0 + iy_0$. Prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$, $u_y = -v_x$ *at* (x_0, y_0). Also prove that $f'(z) = u_x + iv_x$ where u_x *and* v_x are evaluated at (x_0, y_0). [05]

OR

Q.4

(c) If $f'(z) = 0$ everywhere in domain D then prove that $f(z)$ must be constant throughout the domain D. [05]

(d) Prove that $u(x,y) = x^3 - 3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ for $u(x,y)$. Also find corresponding analytic function $f(z)$. [05]

Q.5

(a) Prove that: $\sin^{-1} z = -ilog[iz + \sqrt{1-z^2}]$. Hence find $\sin^{-1} 1$ [05]

(b) Prove that $\sin z = 0$ iff $z = n\pi$, $n \in \mathbb{Z}$. [05]

OR

Q.5

(c) Prove that $\tanh^{-1} z = \dfrac{1}{2} \log \left[\dfrac{1+z}{1-z}\right]$. [05]

(d) Prove the following: [05]
(i) $\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$.
(ii) $2\cos z_1 \sin z_2 = \sin(z_1 + z_2) - \sin(z_1 - z_2)$.

Q.6

(a) Find the image of rectangle $0 \le x \le 1$, $0 \le y \le 2$ under the transformation $w = (i+1)z + 2$. Also sketch rectangle and its image. [05]

(b) Find linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$, $z_3 = i$ onto $w_1 = -1$, $w_2 = i$, $w_3 = 1$. [05]

OR

Q.6

(c) Prove that all linear fractional transformation that maps the upper half plane $Imz > 0$ on to the open disk $|w| < 1$ and the boundary $Imz = 0$ on to the boundary of $|w| = 1$ is given by $w = e^{i\alpha}\left[\dfrac{z - z_0}{z - \overline{z_0}}\right]$, $(Imz_0 > 0)$. [07]

(d) Find the image of the line $\le y = \le 1/2$ under the transformation $w = 1/z$. Show it graphically. [03]

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(2)