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Sardar Patel University, Vallabh Vidyanagar

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Real Analysis - III

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Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] In usual notations, the Cauchy's form of remainder in Taylor's theorem is

[A] $\dfrac{h^n(1-\theta)^n}{n!}f^{(n)}(a+\theta h)$ [B] $\dfrac{h^{n-1}(1-\theta)^n}{n!}f^{(n-1)}(a+\theta h)$

[C] $\dfrac{h^n}{(n-1)!}f^{(n)}(a+\theta h)$ [D] $\dfrac{h^n(1-\theta)^{n-1}}{(n-1)!}f^{(n)}(a+\theta h)$

[2] If a function f defined on $[a,b]$ is continuous on $[a,b]$ and differentiable on (a,b) then the tangent at atleast one point on the curve $y=f(x)$ is

[A] parallel to the X-axis
[B] perpendicular to the X-axis
[C] parallel to the chord joining $(a,f(a))$ and $(b,f(b))$
[D] perpendicular to the chord joining $(a,f(a))$ and $(b,f(b))$

[3] Which of the following functions does not satisfy all the conditions of Rolle's theorem on $[0,1]$?

[A] x^2-x [B] x^3-x^2 [C] x^4-x^3 [D] x^2-1

[4] If f has a minima at c then there is some $\delta > 0$ such that $\forall x \in (c-\delta, c+\delta), x \neq c$

[A] $f(c) < f(x)$ [B] $f'(c) < f'(x)$ [C] $f(c) > f(x)$ [D] $f'(c) > f'(x)$

[5] Number of stationary points of the function $f(x)=x^2-2x-1$ is

[A] 0 [B] 1 [C] 2 [D] 3

[6] A function $f(x)$ has a maximum at c if while x passes through c, f changes from

[A] an increasing to a decreasing function
[B] a decreasing to an increasing function
[C] a decreasing to a constant function
[D] none

[7] If P_0 is a partition of $[a,b]$ and P_1 is a refinement of P_0 and P_2 is a refinement of P_1 then

[A] $L(P_0,f) \leqslant L(P_2,f)$ [B] $L(P_0,f) \geqslant L(P_2,f)$

[C] $L(P_2,f) \leqslant L(P_1,f)$ [D] $L(P_1,f) \leqslant L(P_0,f)$

[8] How many subintervals of the partition $\{1,2,3,6,8,11,14\}$ of $[1,14]$ have their lenths equal to norm of the partition?

[A] 0 [B] 1 [C] 2 [D] 3

(P.T.O.)

[9] If a function f is integrable over $[1,3]$ and $[3,5]$ then $\int_1^5 f.dx =$

[A] $\int_1^3 f.dx - \int_3^5 f.dx$ [B] $\int_1^3 f.dx + \int_3^5 f.dx$ [C] $\int_3^5 f.dx - \int_1^3 f.dx$ [D] none

[10] For a function $f(x) = 2,\ \forall x \in [0,2]$, and any partition P of $[0,2]$, the upper sum $U(P,f) =$

[A] 1 [B] 2 [C] 3 [D] 4

Q: 2. Answer ANY TEN of the following. **20**

[1] Explain the algebraic meaning of Rolle's theorem

[2] Is Rolle's theorem applicable to $f(x) = 2x + 1$ on $[0,2]$? Why?

[3] In usual notations write the Lagrange's and Cauchy's forms of remainders of Maclaurin's expansion.

[4] Show that, $f(x) = x^2 - 4x - 5$ has a minimum at 2

[5] Show that $f(x) = x^3$ has no extreme value at 0

[6] Show that $x = \pm 2$ are stationary points of $f(x) = x^3 - 12x + 1$.

[7] For a bounded function $f(x) = x^2, x \in [1,7]$ and partition $P = \{1,2,5,7\}$, find $L(P,f)$

[8] If $P_1 = \{0,1,2,5\}$, $P_2 = \{0,1,3,5\}$ and $P_3 = \{0,1,2,4,5\}$ are three partition of $[0,5]$ then find a common refinement of P_1 and P_2 which is not a refinement of P_3.

[9] For a function $f(x) = 4x, x \in [1,8]$ and a partition $P = \{1,2,5,8\}$, verify $L(P,f) < U(P,f)$

[10] Evaluate $\int_1^4 [x].dx$, where $[x]$ denotes the greatest integer not exceeding x.

[11] A function f is integrable over $[a,c]$ and $[c,b]$. If $\int_a^c f.dx = 2k$, $\int_c^b f.dx = 4k$ and $\int_a^b f.dx = 24$ then find k.

[12] A function f has infinite number of points of discontinuity but the set of discontinuties has only one limit point in $[2,8]$. Can it be integrable over $[2,8]$? Justify.

Q: 3 [A] State and prove Lagrange's Mean Value theorem **5**

[B] If a function $f(x)$ satisfies the conditions of the Lagrange's Mean Value Theorem on $[a,b]$ then prove the following
(i) If $f'(x) = 0,\ \forall x \in [a,b]$ then f is constant on $[a,b]$
(ii) If $f'(x) > 0,\ \forall x \in [a,b]$ then f is strictly increasing on $[a,b]$ **5**

OR

Q: 3 [A] State and prove Taylor's theorem. 5

[B] A twice differentiable function f is such that $f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove that there is al least one value ξ between a and b for which $f''(\xi) < 0$. 5

Q: 4 [A] If c is an interior point of the domain of a function f and $f'(c) = 0$ then prove that the function has maxima or minima at c according as $f''(c)$ is negative or positive 5

[B] Examine the function $\sin x + \cos x$ for extreme values 5

OR

Q: 4 [A] Define Extreme Value . Also examine the function $(x-3)^5(x+1)^4$ for extreme values 5

[B] Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. 5

Q: 5 [A] State and prove Darboux's Theorem. 5

[B] Prove that a necessary and sufficient condition for the integrability of a bounded function f is that for every $\epsilon > 0$ there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \epsilon$. 5

OR

Q: 5 [A] Prove that if f_1 and f_2 are bounded and integrable functions on $[a, b]$, then their product $f_1 f_2$ is also bounded and integrable on $[a, b]$ 5

[B] Define Riemman Integrable function and show that $3x + 1$ is integrable on $[1, 2]$ and evaluate $\int_1^2 (3x+1)dx$ 5

Q: 6 [A] If a function f is bounded and integrable on $[a, b]$, then show that the function F defined as

$$F(x) = \int_a^x f(t).dt, \ a \leqslant x \leqslant b$$

, is continuous on $[a, b]$. Also, if f is continuous at a point c of $[a, b]$, then prove that F is derivable at c and $F'(c) = f(c)$. 5

[B] If a function f is monotonic on $[a, b]$, then prove that f is integrable on $[a, b]$. 5

OR

Q: 6 [A] Show that a function f is integrable over $[a, b]$ iff for $\epsilon > 0$, there exists $\delta > 0$ such that if P, P' are any two partitions of $[a, b]$ with mesh less than δ then

$$|S(P, f) - S(P', f)| < \epsilon$$

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[B] State and prove the Second Mean Value theorem of Integral Calculus 5

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