[A-93] No. Of Printed Pages: 3

SARDAR PATEL UNIVERSITY

VALLABH VIDYANAGAR

B.SC SEM-VI

SUB: MATHEMATICS **Course No: US06CMTH04 (Abstract Algebra-II)**

DATE: 04/04/2016 **TIME: 02:30 PM TO 05:30 PM**

Marks: 70

Q:1 Select correct options in your answer book. [10]

(1) ___ is a non-commutative ring.

(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) none of these

(2) G.C.D. of 2+3i and 4+7i is____.

(a) i (b) −1 (c) 1 (d) − i

(3) ___ is field.

(a) $\mathbb{Q}$ (b) $\mathbb{Z}$ (c) $M_2(\mathbb{R})$ (d) all of these

(4) Quotient field of ring of Gaussian integer is ___.

(a) $\mathbb{Z}+i\mathbb{Z}$ (b) $\mathbb{Q}+i\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) $\mathbb{Q}$

(5) ___ is sub ring of $\mathbb{Q}$.

(a) 0 (b) $\mathbb{Z}$ (c) $\{\pm 1\}$ (d) $\mathbb{N}$

(6) If I is ideal in ring R then unit element of R/ I is ___.

(a) 0 (b) 1 (c) R (d) 1+ I

(7) Let R be any ring, $f(x) \in R[x], \alpha \in R$ is said to be root of f(x) if f(x)=__

(a) 1 (b) 2 (c) −1 (d) 0

(8) If F is field, $f(x) \in F[x]$, $\alpha \in F$ is a root of f(x) then ____.

(a) $(X-\alpha)$ /f(x) (b) $(x+\alpha)$ /f(x) (c) f(x) $/(x-\alpha)$ (d) f(x) $/(x+\alpha)$

(9) Every ___ is principal ideal domain.

(a) Integral domain (b) ring (c) field (d) Euclidean domain

(10) Every irreducible element in unique factorization domain is____.

(a) unit (b) not unit (c) prime (d) not unit

Q:2 Answer the following in short. (**Attempt Any Ten**) [20]

(1) Define Ring.

(2) Define Embedded ring and Kernal of homomorphism.

(3) For prime p, prove that Z_p is a field.

(4) Find Quotient field of Z.

(5) Prove that {0, 3} is an ideal in Z_6 .

(6) Let f: R→ R′ be any ring homomorphism, then prove that Ker (f) is an ideal in R.

(7) Define Euclidean Domain.

(8) Prove that 1+ i is irreducible in Z+ iZ.

(9) Let R = {a+b$\sqrt{-5}$: a, b∈ Z}, then prove that 1+2$\sqrt{-5}$ and 3 are relatively prime.

(10) Find all roots of X^3+ 5X in Z_6.

(11) If p is prime and n>1 then prove that X^n−p ∈ Z[x] is irreducible.

(12) Find Content of the f(x) = 3X^3−2X^2+ 6X+9 ∈ Z[x] , where R= Z+ iZ.

Q. 3

(a) Show that (Z_n,+, ·) forms a ring. Is it commutative? [05]

(b) Prove that every field is an Integral domain. Does the converse hold? Verify it. [05]

Q. 3

OR

(c) Prove that every finite Integral domain is field. [05]

(d) State and prove Cayley’s theorem for ring. [05]

Q. 4

(a) Prove that every field is simple ring. [05]

(b) State and prove first isomorphism theorem for ring. [05]

Q. 4

OR

(c) Prove that P is prime ideal of $\mathbb{Z}$ iff either P = 0 or P = p$\mathbb{Z}$ for some prime number p. [05]

(d) Prove that an ideal P in commutative ring R is a prime ideal iff $R/_P$ is an integral domain. [05]

Q. 5

(a) Show that the Ring of Gaussian integers is Euclidean domain. [05]

(b) Prove that every Euclidean domain is factorization domain. [05]

Q. 5

OR

(c) Prove that every principal ideal domain is factorization domain. [05]

(d) Prove that every Euclidean domain is principal ideal domain. [05]

Q. 6

(a) State and prove Gauss lemma. [06]

(b) Let R be a unique factorization domain. Then Prove that product of two primitive polynomials over R is also a primitive polynomial. [04]

Q. 6 **OR**

(c) State and prove Eisenstein's criterion. [06]

(d) Let F be afield and $f(x) \in F(x)$ be a polynomial of degree n. Prove that f(x) has at most n distinct roots in F. [04]