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[A 85]

SARDAR PATEL UNIVERSITY

B.Sc. (SEM-VI) Examination(Regular & NC)

Friday, 1st April-2016

USO6CMTHO3 (Topology)

Time: 2:30 P.M. TO 5:30 P.M. **Maximum Marks : 70**

Note: Figures to the right indicate marks to the questions.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

(1) _______ is neither u-open nor u-closed.

(a) (2,9) (b) [2,9] (c) [2,9) (d) none

(2) _______ Intersection of closed set is closed.

(a) Finite (b) Arbitrary (c) Infinite (d) none

(3) Let (X, τ) be a topological space. A subset F of x is τ-closed if X- F ____ τ.

(a) $\not\subset$ (b) $\subset$ (c) $\notin$ (d) $\in$

(4) Two topological spaces are said to be topologically equivalent iff they are ________.

(a) continuous (b) not continuous (c) homeomorphic (d) not homeomorphic

(5) _______ is not a cluster point of (-2, 9) in $(\mathbb{R}, u)$.

(a) -3 (b) -2 (c) 0 (d) 9

(6) In (X, τ), if $A \subset X$ then Int(A) is the _______ subset of A.

(a) Smallest τ-closed (b) largest τ-closed (c) smallest τ-open (d) largest τ-open

(7) Let I_1 and I_2 be any two open intervals then (I_1, u_{I_1}) and (I_2, u_{I_2}) are _________.

(a) Isomorphic (b) Homeomorphic (c)not Homeomorphic (d) none

(8) (2, 5) is _________ in its relativized u-topology.

(a) connected (b) disconnected (c) compact (d) none.

(9) Which of the following is not true for topological spaces?

(a) Every T_3-space is T_2 –space (b)Every T_2-space is T_1-space

(c) $(\mathbb{R}, u)$ is a T_2-space (d) Every T_1-space is T_2-space

(10) Every _______ is compact in its relativized u-topology.

(a) [a, b) (b) (a, b] (c) [a ,b] (d) (a, b)

Q.2 Answer ANY TEN of the following: **[20]**

(1) Show that (-1, 3) ∪ (6, 7) is u- open set.

(2) Prove that {a} is closed set in $(\mathbb{R}, u)$ if $a \in \mathbb{R}$.

(3) Find X and τ for which (X, τ) is not a topological space.

(4) Define: Dense set.

(5) Check whether 1 is an interior point of [0, 1] in $(\mathbb{R}, u)$ or not.

(6) Define: continuous mapping in topological space.

(7) For X={a ,b ,c }, Determine whether $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$ is connected or disconnected.

(8) Let $(\mathbb{R}, u)$ be a topological space. Show that u is a Hausdroff topology for R.

(9) If (X,τ) is a Hausdroff space and $p \in X$ then prove that { p } is τ – closed set.

(10) Define: Regular space

(11) Prove that every T_2–space is T_1–space.

(12) Define: T_1 – space

Q.3

Define: Door Space. Give an example of a space which is door space and one example of a space which is not Door space. Justify your answer. [10]

OR

Q.3 Define: Coarser Topology, Finer Topology, Non-comparable topologies . Give an example of each. Justify your answer. [10]

Q.4

(a) Let (X, τ) be a topological space and $A \subset X$ then prove that Int(A) is τ –open set. [5]

(b) If (X,τ) *and* (Y,ψ) are topological spaces and $f: X \to Y$ is a continuous mapping then prove that inverse image of every ψ – nbhd of f(x) is τ – nbhd of x; $x \in X$. [5]

OR

Q.4

(a) Let (X,τ) *be* a topological space and $A \subset X$ then prove that A is τ -open set iff Int(A) = A. [5]

(b) If (X,τ) *and* (Y,ψ) are topological spaces. If the inverse image of f of every ψ –closed set is τ –closed set then prove that the mapping f *is* continuous. [5]

Q.5

(a) If a topological space (X,τ) has a non-empty proper subset A that is both τ –open and τ –closed then prove that (X,τ) is disconnected. [5]

(b) Prove that a continuous image of a connected space is connected. [5]

OR

Q.5

(a) If a topological space (X,τ) is disconnected then prove that there is a non-empty proper subset of X that is both τ – open and τ –closed. [5]

(b) If (Y,τ_Y) is a compact subspace of a Hausdroff space (X,τ) then prove that Y is τ –closed. [5]

Q.6

(a) Prove that a continuous image of compact space is compact. [5]

(b) Show that the space $(\mathbb{R}, u)$ *is a* T_3–space. [5]

OR

Q.6

(a) Prove that every compact Hausdroff space is a T_3–space . [5]

(b) Show that every metric space is a Hausdroff space. [5]

X=X=X

(2)