(50) **Seat No.:** ________ **No. of Printed Pages : 3**

SARDAR PATEL UNIVERSITY

B.Sc SEM-II (NC) EXAMINATION
22[nd] October 2016 , Saturday
02.00 p.m. to 04.00 p.m.
US02EMTH02
(Mathematics)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The order and degree of differential equation $\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$ is ....

(a) 1, 2 (b) 2, 1 (c) 3, 3 (d) 2, 2

(2) $\frac{d}{dx}(uv) =$ ...... if u and v are functions of x,

(a) $\frac{u\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (b) $\frac{-\frac{dv}{dx}}{u^2}$ (c) $u\frac{dv}{dx}$ (d) $u\frac{dv}{dx} + v\frac{du}{dx}$

(3) $\int \sec x \tan x \, dx =$ ....

(a) $\tan^2 x + c$ (b) $\sec x + c$ (c) $-\sec^2 x + c$ (d) $\tan x + c$

(4) The integration by parts method is stated as $\int uv \, dx =$ ....

(a) $u \int v \, dx - v \int udx$ (b) $u \int v \, dx + v \int udx$
(c) $u \int v \, dx - \int \frac{du}{dx} (\int v \, dx) \, dx$ (d) $u \int v \, dx - v \int u (\int v \, dx) \, dx$

(5) $\lim_{x\to 0} \frac{\sin kx}{3x} = \frac{1}{5}$ then $k =$ ....

(a) 15 (b) $\frac{5}{3}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

(6) $\int f(x) \, dx = f(x) + c$ then $f(x) =$ ....

(a) constant (b) 0 (c) e^x (d) $\log x$

(7) $\int_{\log 2}^{\log 5} e^x \, dx =$ ....

(a) -3 (b) 3 (c) 0 (d) 7

(8) If $f(x) = \int_0^x x \, dx$ then $f(0) + f(1) + f(2) =$ .....

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{5}{2}$ (d) 1

(9) $\lim_{x\to a} \frac{x^n - a^n}{x - a} =$ ....

(a) $(n-1)a^{n-1}$ (b) na^{n+1} (c) na^{n-1} (d) nx^{n-1}

(10) The solution of differential equation $\frac{dy}{dx} = y$ is ....

(a) x (b) 0 (c) e^x (d) None of these

Q.2 Attempt any **ten**: [20]

(1) Find the differential equation of $y = a\sin(x + b)$.

(2) Find $\lim_{x\to 1} \frac{x^6 - 1}{x^{15} - 1}$ $(x \in R - \{1\})$

(3) Find $\frac{d}{dx} \log(\sin x)$.

(4) Find $\int x \log x \, dx$.

(P.T.O)

(5) Find $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x}$.

(6) Evaluate : $\int (x+2)(x+1)\ dx$.

(7) State fundamental principle of definite integration .

(8) Find $\int_{-1}^{1} x\ e^{x}\ dx$

(9) Find the value of $\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x\ dx$.

(10) Define: Differential equation , Order and degree of differential equation .

(11) Find derivative of $y = x^x + sinx^x$.

(12) Solve : $xdy - ydx = 0$

Q.3

(a) If $x^y = e^{x-y}$ then prove that $\dfrac{dy}{dx} = \dfrac{\log x}{[\log xe]^2}$. [5]

(b) Evaluate: $\lim\limits_{x\to 1}\ \dfrac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}$ [5]

OR

Q.3

(c) Obtain derivative of the following function using definition .

(i) $\dfrac{1}{\sqrt{x}}$ (ii) e^{2x} . [5]

(d) If $y = \cos^{-1}\left(\dfrac{3+5\cos x}{5+3\cos x}\right)$ then prove that $\dfrac{dy}{dx} = \dfrac{4}{5+3\cos x}$. [5]

Q.4

(a) Evaluate the following integrations.

(i) $\int \left(e^{a\log x} + e^{x\log a}\right)\ dx$, (ii) $\int \dfrac{1}{\sqrt{2ax - x^2}}\ dx, (0 < x < 2a)$ [5]

(b) Evaluate:

(i) $\int (\sin x + e^x + 4^x + x^4)\ dx,$ (ii) $\int \dfrac{(\log x)^n}{x}\ dx, x > 0$ [5]

OR

Q.4

(c) Evaluate the following integrations.

(i) $\int \dfrac{1}{4x^2+9}\ dx$, (ii) $\int \dfrac{\cos x}{\cos x - 1}\ dx$. [5]

(d) Evaluate:

(i) $\int x^3 \tan^{-1} x\ dx$. (ii) $\int \dfrac{a + b\cos x}{\sin^2 x}\ dx$ [5]

Q.5

(a) If $f(x) = f(a+b-x)$, Prove that $\int\limits_a^b x f(x)\ dx = \dfrac{(a+b)}{2}\int\limits_a^b f(x)\ dx$ [5]

(b) Evaluate: (i) $\int\limits_0^2 \dfrac{6x+3}{x^2+4}\ dx$, (ii) $\int\limits_0^{\pi} \sin^4 x \cos^3 x\ dx$. [5]

OR

Q.5

(c) Evaluate: (i) $\int\limits_0^1 \dfrac{2x+3}{5x^2+1}\ dx$, (ii) $\int\limits_0^{\frac{1}{2}} \dfrac{sin^{-1}x}{(1-x^2)^{3/2}}\ dx$. [5]

(d) Prove that $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}\,dx = a.$ [5]

Q.6

(a) Obtain the differential equation of family of all the parallel lines represented by $y = 2x + c$ with slop 2 .(c is arbitrary constant) [5]

(b) Verify that $y = x^2 + cx$(c is arbitrary constant) is the general solution of the differential equation $xy' = x^2 + y$. [5]

OR

Q.6

(c) Solve $y(1+e^x)dy = (y+1)e^x dx$, then find the particular solution of the given differential equation. [5]

(d) Solve $y(1+e^x)dy = (y+1)e^x dx$. [5]

— x —

(3)