(10) **No. of Printed Pages : 2**

SARDAR PATEL UNIVERSITY

B.Sc. (*SEMESTER – II*) (NC) EXAMINATION-2016

October 15, 2016, Saturday

2.00 p.m. to 4.00 p.m.

US02CMTH02(Matrix Algebra and Differential Equations)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) If a matrix is of 4×6 order then each column has....
(a) 4 elements (b) 6 elements (c) 24 elements (d) 10 elements

(2) A symmetric matrix with all real entries is....
(a) Scalar matrix (b) Hermitian (c) Skew Hermitian (d) Skew symmetric

(3) If 3 is a characteristic root of A, then $A - 3I$ is
(a) a singular matrix (b) a non-singular matrix
(c) an orthogonal matrix (d) none of these

(4) If $|A + 4I| = 0$, then a eigenvalue of A is....
(a) 1 (b) 4 (c) -4 (d) -1

(5) Characteristic equation of the identity matrix I of order 2 is...
(a) $x^2 + 1 = 0$ (b) $x^2 - 1 = 0$ (c) $(x-1)^2 = 0$ (d) $(x+1)^2 = 0$

(6) The complementary function of $(D^2 - 4)y = \sin(x)$ is
(a) $c_1e^{2x} + c_2e^{-2x}$ (b) $c_1e^{x} + c_2e^{-x}$ (c) $(c_1 + c_2x)e^{2x}$ (d) $c_1 \sin 2x + c_2 \cos 2x$

(7) The particular integral of $(D-3)^2y = e^{3x}$ is....
(a) $\frac{x^3}{3!}e^{3x}$ (b) $\frac{x^2}{2!}e^{3x}$ (c) $\frac{x^3}{2!}e^{3x}$ (d) $\frac{x^2}{3!}e^{3x}$

(8) If $f(m) \neq 0$, the particular integral of $f(D)y = e^{mx}$ is....
(a) $\frac{1}{f(-m^2)}e^{mx}$ (b) $\frac{1}{f(-m)}e^{mx}$ (c) $\frac{1}{f(m)}e^{mx}$ (d) $\frac{1}{f(-m)}e^{-mx}$

(9) The particular integral of $(D^2 + m^2)y = \sin mx$ is....
(a) $\frac{-\sin mx}{2m^2}$ (b) $\frac{-x\cos mx}{2m}$ (c) $\frac{\sin mx}{2m^2}$ (d) $\frac{x\cos mx}{2m}$

(10) The particular integral of $(D^4 + D^2)y = \cos 2x$
(a) $1/8 \sin 2x$ (b) $-1/8 \cos 2x$ (c) $1/12 \cos 2x$ (d) $-1/12 \cos 2x$

Q.2 Attempt any Ten: [20]

(1) Define: (i) Symmetric Matrix (ii) Diagonal Matrix.

(2) If A is a symmetric matrix, then show that A^3 is also a symmetric matrix.

(3) Explain why in general $A^2 - B^2 \neq (A-B)(A+B)$.

(4) Define Characteristic roots and Characteristic vectors of a matrix .

(5) Find the characteristic roots of $\begin{bmatrix} 4 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(6) Find characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -4 & 0 \\ 3 & 2 & 6 \end{bmatrix}$.

(7) Let y_1 and y_2 be two solutions of a linear differential equation $\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_ny = 0$ and C_1, C_2 be two arbitrary constants. Then prove that $C_1y_1 + C_2y_2$ is also its solution .

(8) Find complimentary function of the differential equation $(D^2 - 4D + 4)y = e^x$.

(9) Find the particular integral of $(D^3 + D - 7)y = e^{2x}$.

(10) Find the particular integral of $(D^3 - 1)y = x^4$.

(11) Find complimentary function of the differential equation $(D^2 - 10D + 21)y = \sin x$.

(12) Find the particular integral of $(D^4 - D^2 - 1)y = \cos 2x$.

(P.T.O.)

Q.3

(a) Prove that every square matrix can be expressed in one and only one way as the sum of a symmetric and skew-symmetric matrix. [05]

(b) State and prove distributive law for matrices. [05]

OR

Q.3

(c) State and prove reversal law for the transpose of a product of matrices. Also show that $(AB)^{\Theta} = B^{\Theta}A^{\Theta}$. [05]

(d) If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then find out the values of α, β such that $(\alpha I + \beta A)^2 = A$. [05]

Q.4

(a) Show that the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence or otherwise obtain the value of A^{-1}. [05]

(b) If S is a real skew-symmetric matrix then prove that $I - S$ is non-singular and the matrix $A = (I+S)(I-S)^{-1}$ is orthogonal. [05]

OR

Q.4

(c) State and prove Cayley-Hamilton theorem. [05]

(d) Find the characteristic roots and characteristic vector corresponding to at least one characteristic root of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. [05]

Q.5

(a) Solve: $(D^2 - 5D + 4)y = 0$. [04]

(b) Obtain rule for finding the particular integral of $f(D)y = e^{mx}$ where m is constant. [06]

OR

Q.5

(c) Solve: $(D^2 + 4)y = \sec 2x$. [05]

(d) Solve $(D^3 - 5D^2 + 7D - 3)y = \cosh x$. [05]

Q.6

(a) In usual notation prove that $\frac{1}{f(D)}\, xV = \left[x - \frac{1}{f(D)} f'(D)\right] \frac{1}{f(D)} V$, where V is a function of x. [05]

(b) Solve $(D^2 + 3D + 2)y = \cos 3x$. [05]

OR

Q.6

(c) Solve $(D^2 + 9)y = x \sin x$. [05]

(d) In usual notation prove that $\frac{1}{f(D)}\, e^{ax}V = e^{ax} \frac{1}{f(D+a)} V$, where V is a function of x. [05]

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