

SARDAR PATEL UNIVERSITY  
B.Sc.(SEMESTER - II )(NC) EXAMINATION - 2019  
Tuesday , 2<sup>nd</sup> April , 2019  
MATHEMATICS : US02EMTH02  
( Mathematics )

Time : 2:00 p.m. to 4:00 p.m.

Maximum Marks : 70

Que.1 Fill in the blanks.

10

(1) If  $\lim_{x \rightarrow 1} f(x) = 5$  then  $\lim_{x \rightarrow 1} [x + x^2 + 2x^3 + f(x)] = \dots\dots\dots$   
 (a)  $3 + f(x)$       (b)  $4 + f(x)$       (c) 9      (d) 8

(2)  $\frac{d}{dx}(\sec x) = \dots\dots\dots$   
 (a)  $\sec^2 x$       (b)  $\sec^2 x \tan x$       (c)  $\sec x \tan x$       (d) None of the above

(3)  $\lim_{x \rightarrow 0} \frac{\sin kx}{3x} = \frac{1}{5}$  then  $k = \dots\dots\dots$   
 (a) 15      (b)  $\frac{5}{3}$       (c)  $\frac{1}{5}$       (d)  $\frac{3}{5}$

(4) If  $\int f(x) dx = \frac{x^{n+1}}{n+1}$  then  $f(x) = \dots\dots\dots$   
 (a)  $x^{n+1}$       (b)  $x^n$       (c)  $(n+1)x^{n+1}$       (d) None of the above

(5)  $\int \frac{1}{x(1+x)} dx = \dots\dots\dots + c$   
 (a)  $\log(x) - \log(1+x)$       (b) 1      (c)  $\log(x) - \log(1-x)$       (d) 0

(6)  $\int |x^2| dx = \dots\dots\dots + c, x \in (-1, 1)$   
 (a)  $\frac{x^3}{3}$       (b)  $\frac{|x|^3}{3}$       (c) does not exist      (d) None of these

(7)  $\int_0^{\pi/4} \sec^2 x dx = \dots\dots\dots$   
 (a) -1      (b) 0      (c) 1      (d) None of the above

(8) If  $f(x) = \int_0^x x dx$  then  $f(0) + f(1) + f(2) = \dots\dots\dots$   
 (a) 0      (b) 1/2      (c) 5/2      (d) None of the above

(9) Which of the following differential equations is of first order and first degree.  
 (a)  $x^2 \frac{d^2y}{dx^2} = 1$       (b)  $\frac{d^2y}{dx^2} + 2y = x^2$       (c)  $\frac{dy}{dx} = 5y$       (d) None of the above

(10) If  $a, b, c$  are arbitrary constant then order of the differential equation where solution is  $y = a \cos x + b \sin x + ce^{-x}$  is  $\dots\dots\dots$   
 (a) 1      (b) 2      (c) 3      (d) None of these

(1) Evaluate  $\lim_{x \rightarrow 1} \frac{x^6 - 1}{x^{15} - 1}$  ( $x \in R - \{1\}$ )

(2) Evaluate  $\frac{d}{dx} (\sin^{-1} \frac{x}{a})$ .

(3) Find  $\frac{dy}{dx}$  for  $x^3 + y^3 = 3axy$ .

(4) Evaluate  $\int x e^x dx$ .

(5) Evaluate  $\int \frac{x^3 - 3x^2 - 4}{\sqrt{x}} dx$ ,  $x > 0$ .

(6) Evaluate  $\int (\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax) dx$ .

(7) State working rules of definite integration.

(8) Evaluate  $\int_0^1 (x^2 + 3) dx$ .

(9) Evaluate  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x dx$ .

(10) Obtain the order and degree of  $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$ .

(11) Obtain the differential equation of family of all the parallel lines represented by  $y = 2x + c$  with slope 2. (c is arbitrary constant)

(12) Solve  $\frac{dy}{dx} = \sin(x + y)$ .

Que.3 (a) Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x + 2} - \sqrt{3x - 2}}$ .

5

(b) Evaluate  $\frac{d}{dx} \left( \frac{\tan 3x}{3^x} \right)$ .

5

OR

Que.3 (c) Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ .

5

(d) If  $y = \cos^{-1} \left( \frac{3 + 5 \cos x}{5 + 3 \cos x} \right)$  then prove that  $\frac{dy}{dx} = \frac{4}{5 + 3 \cos x}$ .

5

Que.4 (a) Evaluate  $\int \frac{1 - \tan x}{1 + \tan x} dx$ .

5

(b) Evaluate  $\int \frac{\cos x}{\cos x - 1} dx$

5

OR

Que.4 (c) Evaluate  $\int \frac{1}{\sqrt{2ax - x^2}} dx$  ( $0 < x < 2a$ ).

5

(d) Evaluate  $\int x \log x dx$

5

Que.5 (a) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta}{(1 + \cos \theta)^2} d\theta$ .

5

(b) Evaluate  $\int_0^{\sqrt{2}} x^3 e^{x^2} dx$ .

5

OR

Que.5 (c) Evaluate  $\int_0^{\frac{\pi}{4}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$  . 5

(d) If  $\int_0^k \frac{\tan x}{1 + \tan x} dx = \frac{\pi}{4}$  , then find  $k$ . 5

Que.6 (a) Verify that  $y = ax + a^2$  ( $a$  is arbitrary constant) is the general solution of the differential equation  $\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) = y$ . 5

(b) Solve the differential equation  $\frac{dy}{dx} = e^{x+y}$ . Find the particular solution subject to initial condition ,  $y(1) = 1$  . Also find  $y(-1)$  . 5

OR

Que.6 (c) Verify that  $y = cx + \frac{1}{c}$  is the general solution of the differential equation

$y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 1$  , where  $c$  is arbitrary constant. 5

(d) Solve the differential equation  $xdy + ydx = xydx, y(1) = 1$  . 5

— X —



[GG/A-20]  
GU

SEAT No. \_\_\_\_\_

No of printed pages : 3

SARDAR PATEL UNIVERSITY  
B.Sc.(SEMESTER - II )(NC) EXAMINATION - 2019  
Tuesday , 2<sup>nd</sup> April , 2019  
MATHEMATICS : US02EMTH02  
( Mathematics )

Time : 2:00 p.m. to 4:00 p.m.

Maximum Marks : 70

Que.1 યોગ્ય વિકલ્પનો ઉપયોગ કરી ખાલી જગ્યા પૂરો .

10

(1) જો  $\lim_{x \rightarrow 1} f(x) = 5$  હોય તો  $\lim_{x \rightarrow 1} [x + x^2 + 2x^3 + f(x)] = \dots\dots$  થાય .  
(a)  $3 + f(x)$  (b)  $4 + f(x)$  (c) 9 (d) 8

(2)  $\frac{d}{dx}(\sec x) = \dots\dots$   
(a)  $\sec^2 x$  (b)  $\sec^2 x \tan x$  (c)  $\sec x \tan x$  (d) આમાથી કોઈ પણ નહીં

(3) જો  $\lim_{x \rightarrow 0} \frac{\sin kx}{3x} = \frac{1}{5}$  હોય તો  $k = \dots\dots$  થાય .  
(a) 15 (b)  $\frac{5}{3}$  (c)  $\frac{1}{5}$  (d)  $\frac{3}{5}$

(4) જો  $\int f(x) dx = \frac{x^{n+1}}{n+1}$  હોય તો  $f(x) = \dots\dots$  થાય .  
(a)  $x^{n+1}$  (b)  $x^n$  (c)  $(n+1)x^{n+1}$  (d) આમાથી કોઈ પણ નહીં

(5)  $\int \frac{1}{x(1+x)} dx = \dots\dots + c$   
(a)  $\log(x) - \log(1+x)$  (b) 1 (c)  $\log(x) - \log(1-x)$  (d) 0

(6)  $\int |x^2| dx = \dots\dots + c, x \in (-1, 1)$   
(a)  $\frac{x^3}{3}$  (b)  $\frac{|x|^3}{3}$  (c) શક્ય નથી (d) આમાથી કોઈ પણ નહીં

(7)  $\int_0^{\pi/4} \sec^2 x dx = \dots\dots$   
(a) -1 (b) 0 (c) 1 (d) આમાથી કોઈ પણ નહીં

(8) જો  $f(x) = \int_0^x x dx$  હોય તો  $f(0) + f(1) + f(2) = \dots\dots$   
(a) 0 (b) 1/2 (c) 5/2 (d) આમાથી કોઈ પણ નહીં

(9) નિચેનામાથી કયા વિકલ સમીકરણ ની એક કક્ષા (order) અને એક પરિમાણ (degree) છે .  
(a)  $x^2 \frac{d^2 y}{dx^2} = 1$  (b)  $\frac{d^2 y}{dx^2} + 2y = x^2$  (c)  $\frac{dy}{dx} = 5y$  (d) આમાથી કોઈ પણ નહીં

(10) જો  $a, b, c$  કોઈ અચળાંક હોય તો જેનો ઉકેલ  $y = a \cos x + b \sin x + ce^{-x}$  હોય એવા વિકલ સમીકરણ ની કક્ષા (order)..... થાય .  
(a) 1 (b) 2 (c) 3 (d) આમાથી કોઈ પણ નહીં

(P.T.O)

Que.2 નીચેના પ્રશ્નના જવાબ આપો. ( કોઈ પણ દસ )

20

(1) શોધો :  $\lim_{x \rightarrow 1} \frac{x^6 - 1}{x^{15} - 1}$  ( $x \in R - \{1\}$ )

(2) શોધો :  $\frac{d}{dx} (\sin^{-1} \frac{x}{a})$ .

(3)  $x^3 + y^3 = 3axy$  માટે  $\frac{dy}{dx}$  શોધો.

(4) શોધો :  $\int x e^x dx$ .

(5) શોધો :  $\int \frac{x^3 - 3x^2 - 4}{\sqrt{x}} dx$ ,  $x > 0$ .

(6) શોધો :  $\int (\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax) dx$ .

(7) નિયત સંકલન માટે ના નિયમો લખો.

(8) શોધો :  $\int_0^1 (x^2 + 3) dx$ .

(9) શોધો :  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x dx$ .

(10)  $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$  વિકલ સમીકરણની કક્ષા (order) અને પરિમાણ (degree) શોધો..

(11)  $y = 2x + c$  થી મળતી બધી સમાંતર રેખાઓ જેનો ઢાળ 2 હોય એવા વિકલ સમીકરણ શોધો, જ્યાં  $c$  આયાણિક છે.

(12) ઉકેલ શોધો :  $\frac{dy}{dx} = \sin(x + y)$ .

Que.3 (a) શોધો :  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x + 2} - \sqrt{3x - 2}}$ .

5

(b) શોધો :  $\frac{d}{dx} \left( \frac{\tan 3x}{3^x} \right)$ .

5

OR

Que.3 (c) શોધો :  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ .

5

(d) જો  $y = \cos^{-1} \left( \frac{3 + 5 \cos x}{5 + 3 \cos x} \right)$  હોય તો સાબિત કરો કે  $\frac{dy}{dx} = \frac{4}{5 + 3 \cos x}$  થાય.

5

Que.4 (a) શોધો :  $\int \frac{1 - \tan x}{1 + \tan x} dx$ .

5

(b) શોધો :  $\int \frac{\cos x}{\cos x - 1} dx$

5

OR

Que.4 (c) શોધો :  $\int \frac{1}{\sqrt{2ax - x^2}} dx$  ( $0 < x < 2a$ ).

5

(d) શોધો :  $\int x \log x dx$

5

Que.5 (a) શોધો :  $\int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta}{(1 + \cos \theta)^2} d\theta$ .

5

(b) શોધો :  $\int_0^{\sqrt{2}} x^3 e^{x^2} dx$ .

5

OR

(2)

Que.5 (c) શોધો :  $\int_0^{\frac{\pi}{4}} \frac{dx}{4 \sin^2 x + 5 \cos^2 x}$

5

(d) જો  $\int_0^k \frac{\tan x}{1 + \tan x} dx = \frac{\pi}{4}$  હોય તો  $k$  શોધો .

5

Que.6 (a) સાબિત કરો કે  $\left(\frac{dy}{dx}\right)^2 + x \left(\frac{dy}{dx}\right) = y$  વિકલ સમીકરણ નો ઉકેલ  $y = ax + a^2$  થાય , જ્યાં  $a$  આયાજાંક છે .

5

(b)  $\frac{dy}{dx} = e^{x+y}$  નો ઉકેલ શોધો . જો શરૂઆત ની શરત  $y(1)=1$  હોય તો વિશિષ્ટ ઉકેલ શોધો . વધુમાં  $y(-1)$  શોધો .

5

OR

Que.6 (c) સાબિત કરો કે  $y \left(\frac{dy}{dx}\right) = x \left(\frac{dy}{dx}\right)^2 + 1$  વિકલ સમીકરણ નો ઉકેલ  $y = cx + \frac{1}{c}$  થાય , જ્યાં  $c$  આયાજાંક છે .

5

(d) ઉકેલ શોધો :  $xdy + ydx = xydx, y(1) = 1$  .

5

— X —

(3)

(3)

