SARDAR PATEL UNIVERSITY
F. Y. B.Sc. (II SEM.) (CBCS) EXAMINATION
Tuesday, 10th April 2012
11.00 am - 1.00 pm
US02CMTH02: Mathematics
(Matrix Algebra and Differential Equations)

Total Marks: 70

Q.1 Choose the correct option for the following questions and write it down in the Answer-sheet. [10]

(1) Product AB of two matrices can be defined only if _______.
(a) They have same order.
(b) Number of rows of A and number of columns of B are equal.
(c) Number of columns of A and number of rows of B are equal.
(d) Both are square matrices.

(2) If $\begin{bmatrix} 2x-1 & 3y \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 2 & 0 \end{bmatrix}$, then x= _______.
(a) 0 (b) -1 (c) 1 (d) 2

(3) Any diagonal matrix is called a scalar matrix if all the diagonal elements are _______.
(a) equal (b) zero (c) one (d) different

(4) $\begin{bmatrix} 6 & -7 \\ 12 & -14 \end{bmatrix}$ is _______ matrix.
(a) orthogonal (b) singular (c) non-singular (d) none of these

(5) If 3 is the characteristic root of A, then _______.
(a) $|I + 3A| = 0$ (b) $|I - 3A| = 0$ (c) $|A + 3I| = 0$ (d) $|A - 3I| = 0$

(6) A square matrix A is said to be orthogonal matrix if ________.
(a) $A \cdot A' = I$ (b) $A \cdot A^{-1} = I$ (c) $A \cdot A^{\Theta} = I$ (d) $A = A'$

(7) $\frac{1}{D^2+4} e^{-2x} =$ ________.
(a) $\frac{x^2}{2!} e^{-2x}$ (b) $-\frac{x^2}{2!} e^{-2x}$ (c) $\frac{1}{8} e^{-2x}$ (d) $-\frac{1}{8} e^{-2x}$

(8) A complementary function of $(D^2 - 4D + 4)\, y = x$ is ________.
(a) $(c_1 + c_2)\, e^{2x}$ (b) $(c_1 + c_2 x)\, e^{2x}$
(c) $c_1 \cos 2x + c_2 \sin 2x$ (d) $(c_1 \cos 2x + c_2 \sin 2x)\, e^{2x}$

(9) $\frac{1}{f(D)} e^{3x} \cdot x^2 =$ _______.
(a) $e^{2x} \frac{1}{f(D+2)} x^2$ (b) $e^{3x} \frac{1}{f(D+3)} x^2$
(c) $e^{2x} \frac{1}{f(D-2)} x^2$ (d) $e^{3x} \frac{1}{f(D-3)} x^2$

(10) $\frac{1}{f(D)} x \sin x =$ _______.

(a) $\left[x-\frac{1}{f'(D)}f(D)\right]\frac{1}{f'(D)}\sin x$ (b) $\left[x+\frac{1}{f'(D)}f(D)\right]\frac{1}{f'(D)}\sin x$

(c) $\left[x-\frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}\sin x$ (d) $\left[x+\frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}\sin x$

Q.2 Answer the following questions in short. **(Attempt Any Ten)** [20]

(1) Show that $A=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$ is an orthogonal matrix.

(2) Define Scalar Matrix with two illustrations.

(3) Define Identity Matrix with two illustrations.

(4) Define Singular Matrix with an illustration.

(5) If $A=\begin{bmatrix}1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1\end{bmatrix}$ then find the eigen values of A.

(6) Define Characteristic Matrix with an illustration.

(7) Solve $(D^3 - 4D^2 + 5D - 2)\, y = 0$.

(8) Prove that $\frac{1}{D-\alpha}x = e^{\alpha x}\int xe^{-\alpha x}\,dx$.

(9) Define Linear Differential Equation with constant coefficients. Give two illustrations.

(10) Find P. I. for $(D^2 + 3D + 2)\, y = \sin 3x$.

(11) Write down the rules for finding the particular integral of f(D)y=cosmx, where m is constant.

(12) Find C. F. for $(D^2 + 2)\, y = (x^2 + 1)\, e^{3x} + e^x \cos 2x$.

Q.3

(a) State and prove associative law for product of matrices. [05]

(b) Prove that every square matrix can be expressed in one and only one way as P + iQ, where P and Q are Hermitian matrices. [05]

OR

Q.3

(a) In usual notations prove that A (B + C) = AB + AC. [05]

(b) If $A=\begin{bmatrix}3 & -4 \\ 1 & -1\end{bmatrix}$, then show that $A^k=\begin{bmatrix}1+2k & -4k \\ k & 1-2k\end{bmatrix}$, where k is a positive integer. [05]

Q.4

(a) State and prove Cayley-Hamilton theorem. [05]

(b) Verify Cayley-Hamilton theorem for the matrix [05]

$A=\begin{bmatrix}1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1\end{bmatrix}$. Also compute A^{-1}.

OR

Q.4

(a) Prove that every orthogonal matrix A can be expressed as $A = (I + S)(I - S)^{-1}$ by a suitable choice of a real skew-symmetric matrix S, provided that -1 is not a characteristic root of A. [05]

(b) Find the characteristic roots and one of the corresponding characteristic vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ [05]

Q.5

(a) Solve: $(D^2 - 3D + 5)\,y = e^{-x}$ [05]

(b) Solve: $(D^2 + 4)\,y = \sec 2x$ [05]

OR

Q.5

(a) Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$, where m is constant. [05]

(b) Solve: $(D^3 - 1)\,y = (e^x - 1)^2$ [05]

Q.6

(a) In usual notations prove that $\frac{1}{f(D)} xV = \left[x - \frac{1}{f(D)} f'(D)\right] \frac{1}{f(D)} V$, where V is a function of x. [05]

(b) Solve: $(D^2 + 3D + 2)\,y = \cos 3x$ [05]

OR

Q.6

(a) Obtain the rule for finding the particular integral of $f(D)y = \sin mx$, where m is constant. [05]

(b) Solve: $(D^3 - D^2 - 6D)\,y = x^3$ [05]

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