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SARDAR PATEL UNIVERSITY
F.Y. B.Sc. (II SEM.) (CBCS) EXAMINATION
2011
Tuesday, 26th April
3.00 pm to 5.00 pm
US02CMTH02 : MATHEMATICS
[Matrix Algebra and Differential Equations]

Total Marks : 70

Note : Figures to the right indicate full marks.

Q.1 Answer the following by selecting correct choice from the options : [10]

[1] $A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a _____ matrix

[a] Scalar [b] Zero
[c] Indentity [d] None of these.

[2] The elements of the principal diagonal of a real skew symmetric matrix are all equal to _____

[a] -1 [b] 1 [c] 0 [d] none of these

3] Total number of elements in (3x4) matrix is _____

[a] 3 [b] 7 [c] 4 [d] 12

4] If $|A - 7I| = 0$ then one of the characteristic roots of A is _____

[a] 1 [b] 7 [c] 0 [d] -7

5] A square matrix A is said to be an orthogonal matrix if _____

[a] $AA^T = I$ [b] $AA^{\Theta} = I$
[c] $AA^{-1} = I$ [d] $A^T = A$

6] A square matrix A is said to be singular if _____

[a] $|A| \neq 0$ [b] $|A| > 0$
[c] $|A| = 0$ [d] $|A| < 0$

7] The general solution of the linear differential equation is _____

[a] x = C.F. + P.I. [b] y = C.F. + P.I.
[c] x = C.F. - P.I. [d] y = C.F. - P.I.

8] For $f(D)y = e^{mx}$, when m is not a root of f(D) = 0, then P.I. = _____

[a] $\frac{1}{\phi(m)} e^{mx}$ [b] $\frac{1}{f(m)} e^{mx}$

[c] $\frac{1}{\phi(m)} \frac{x^r}{r!} e^{mx}$ [d] $\frac{1}{f(m)} \frac{x^r}{r!} e^{mx}$

9] $\frac{1}{f(D)}e^{2x}\sin x =$ ______

[a] $e^{2x}\frac{1}{f(D)}\sin x$ [b] $\frac{1}{f(D+a)}e^{2x}\sin x$

[c] $e^{2x}\frac{1}{f(D-2)}\sin x$ [d] $e^{2x}\frac{1}{f(D+2)}\sin x$

10] $\frac{1}{f(D)}x\cos 2x =$ ______

[a] $\left[x-\frac{1}{f'(D)}f(D)\right]\frac{1}{f'(D)}\cos 2x$ [b] $\left[x+\frac{1}{f'(D)}f(D)\right]\frac{1}{f'(D)}\cos 2x$

[c] $\left[x-\frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}\cos 2x$ [d] $\left[x+\frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}\cos 2x$

Q.2 Attempt **any Ten** : [20]

[1] Define upper triangular matrix with two illustrations.

[2] If A and B both are symmetric, then prove that AB is symmetric iff A and B commute.

[3] Prove that $(AB)^{\theta} = B^{\theta} A^{\theta}$

[4] Define characteristic matrix with an illustration.

[5] Find the characteristic equation of

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 2 & 0 \end{bmatrix}$$

[6] Define Non-singular matrix with two illustrations.

[7] Let y_1 and y_2 be two solutions of a linear differential equation $\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + ... + a_n y = 0$; c_1, c_2 be two arbitrary costants. Then prove that $c_1y_1 + c_2y_2$ is also a solution.

[8] Find C.F. for $(D^2 - 5D + 4)y = 0$

[9] Define Linear Differential Equation with constant coefficients. Give two illustrations.

[10] Find P.I. for $(D^2 + 3D + 2)y = \cos 3x$

[11] Find C.F. for $(D^3 + 4D)y = \sin 2x + 3e^{2x} + 2x$

[12] Write down the rule for finding the particular integral of $f(D)y = \sin mx$, where m is constant.

Q.3

[a] State and prove reversal law for the transpose of a product of matrices. [05]

[b] Prove that every square matrix can be expressed in one and only one way as P + iQ where P and Q are Hermitian matrices. [05]

OR

Q.3

[a] State and prove distributive law for matrices. [05]

[b] Prove that every square matrix can be expressed in one and only one way as the sum of symmetric and skew-symmetric matrices. [05]

Q.4

[a] State and prove Cayley-Hamilton theorem. [05]

[b] Find the characteristic roots and one of the corresponding characteristic vector for the matrix [05]

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

Q.4

[a] If S is a real shew-symmetric matrix then prove that I-S is non-singular and the matrix $A = (I+S)(I-S)^{-1}$ is orthogonal. [05]

[b] Show that the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley - Hamilton theorem. [05]

Q.5

[a] Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$, where m is constant. [05]

[b] Solve, $(D^2 + a^2)y = \text{cosec } ax$ [05]

OR

Q.5

[a] In usual notations prove that, [05]

$$\frac{1}{D-\alpha} X = e^{\alpha x} \int e^{-\alpha x} X \, dx$$

[b] Solve, $(D^3 - 5D^2 + 7D - 3)\, y = \cosh x$ [05]

Q.6

[a] In usual notations prove that, [05]

$$\frac{1}{f(D)} e^{ax} v = e^{ax} \frac{1}{f(D+a)} v$$

[b] $(D^2 + 9)\, y = x \sin x$ [05]

OR

Q.6

[a] Obtain the rule for finding the particular integral of $f(D)y = \sin mx$, where m is constatn. [05]

[b] Solve, $(D^2 - 2D + 1)\, y = x^2 e^{3x}$ [05]

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