Sc

[72 & A-31 ENG]

SEAT No._____ **No. of Printed Pages : 3**

SARDAR PATEL UNIVERSITY

B.Sc SEM-II EXAMINATION

06^{th} April 2018 , Friday

02.00 p.m. to 04.00 p.m.

US02EMTH02

(Mathematics)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) If f is even function then $f(-x) =$ .....

(a) $f(x)$ (b) $-f(x)$ (c) 2 (d) None of these

(2) $\int x + e^x \, dx =$ .....

(a) $x^2 + e^x + c$ (b) $x + e^x + c$ (c) $\frac{x^2 + 2e^x}{2} + c$ (d) $\frac{x + e^x}{2} + c$

(3) For any point on circle $x^2 + y^2 = a^2$, $\int (x^2 + y^2) \, dx =$ .....

(a) $ax + c$ (b) 0 (c) $a^2x + c$ (d) None of these

(4) If $a < c < b$ then $\int_a^c f(x) \, dx + \int_c^a f(x) \, dx =$ .....

(a) $2\int_a^c f(x) \, dx$ (b) $2\int_c^b f(x) \, dx$ (c) $2\int_a^b f(x) \, dx$ (d) None of these

(5) $\int_0^1 xe^x \, dx =$ .....

(a) -1 (b) 1 (c) 0 (d) None of these

(6) The order and degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$ is .....

(a) 2, 1 (b) 1, 2 (c) 3, 1 (d) 2, 3

(7) $\lim_{x \to 0} \sin x =$ .....

(a) 1 (b) 0 (c) -1 (d) None of these

(8) The solution of differential equation $\frac{dy}{dx} = y$ is .....

(a) x (b) 0 (c) e^x (d) None of these

(9) $\int (x-a)(x-b)(x-c)...(x-z) \, dx =$ .....

(a) Polynomial function of degree 5 (b) Polynomial function of degree 7
(c) constant function (d) none of these

(10) $\frac{d}{dx} \tan^{-1} x =$ .....

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $-\frac{1}{1+x^2}$

Q.2 Attempt any **ten**: [20]

(1) Evaluate : $\int \frac{x^2}{1+x^6} \, dx$.

(2) Solve : $xdy - ydx = 0$.

(3) Find $\frac{d}{dx} \log(\sin x)$.

[PTO]

(4) Find the value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x \, dx$.

(5) Find $\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x}$.

(6) Find derivative of $x^2y + y^2x = e^{x+y}$.

(7) State fundamental principle of definite integration .

(8) Find the differential equation of $y = a\sin(x + b)$.

(9) Define: Differential equation , Order and degree of differential equation .

(10) If $\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists, find a.

(11) Evaluate : $\int_{\log_a 4}^{\log_a 2} a^x \, dx$.

(12) Find $\int x \cos x \, dx$.

Q.3

(a) Obtain derivative of the following function using definition . [5]

(i) $\sqrt{x}$ (ii) e^{3x} .

(b) If $y = \cos^{-1}\left(\frac{3 + 5\cos x}{5 + 3\cos x}\right)$ then prove that $\frac{dy}{dx} = \frac{4}{5 + 3\cos x}$. [5]

OR

Q.3

(c) Find the following.

(i) $\frac{d}{dx}\left((\sin x)^x + x^{\cos x}\right)$

(ii) $\frac{d}{dx}\left(x^{\sqrt{x}} + (\sqrt{x})^x\right) \quad , x > 0$ [5]

(d) Evaluate: $\lim_{x \to 2} \frac{x^2 - 4}{\sqrt{x + 2} - \sqrt{3x - 2}}$ [5]

Q.4

(a) Evaluate the following integrations. [5]

(i) $\int \frac{1}{\sqrt{2x^2 + 3}} \, dx$, (ii) $\int \frac{dx}{x^2\sqrt{1 - x^2}}$.

(b) Evaluate: [5]

(i) $\int \frac{\sqrt{3 - x}}{x} \, dx \quad x \in (0, 3)$, (ii) $\int \frac{(\log x)^n}{x} \, dx, x > 0$

OR

Q.4

(c) Evaluate the following integrations. [5]

(i) $\int \frac{x^3 + 3x^2 + 4}{\sqrt{x}} \, dx \quad , x > 0$, (ii) $\int \frac{\cos x}{\cos x - 1} \, dx$

(d) Evaluate: [5]

(i) $\int x^3 \tan^{-1} x \, dx$. (ii) $\int \frac{\cos 2x}{\sin^2 2x} \, dx$

Q.5

(a) Evaluate: (i) $\int_0^2 \frac{6x + 3}{x^2 + 4} \, dx$, (ii) $\int_0^{\frac{1}{2}} \frac{sin^{-1}x}{(1 - x^2)^{3/2}} \, dx$. [5]

(b) If $\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$, then find k. [5]

OR

Q.5

(c) If $f(x) = f(a+b-x)$, Prove that $\int_a^b xf(x)\,dx = \frac{(a+b)}{2}\int_a^b f(x)\,dx$ [5]

(d) Evaluate: (i) $\int_0^{\frac{\pi}{4}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$, (ii) $\int_0^{\pi} \sin^4 x \cos^3 x\ dx$. [5]

Q.6

(a) Solve the differential equations $\frac{dy}{dx} = e^{x+y}$.Find the particular solution subject to initial condition , $y(1) = 1$. Also find $y(-1)$. [5]

(b) Verify that $y = x^2 + cx$(c is arbitrary constant) is the general solution of the differential equation $xy' = x^2 + y$. [5]

OR

Q.6

(c) Obtain the differential equation of family of circles having centre on x - axis and radius 1 unit. [5]

(d) Solve the differential equations $x(1+y^2)dx - y(1+x^2)dy = 0$. [5]