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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - II) EXAMINATION - 2016
Tuesday , 5^{th} April , 2016
MATHEMATICS : US02EMTH02
(Mathematics)

Time : 10:30 a.m. to 12:30 p.m. **Maximum Marks : 70**

Que.1 Fill in the blanks. 10

(1) If $\lim_{x\to 1} f(x) = 5$ then $\lim_{x\to 1}[x + x^2 + 2x^3 + f(x)]$ = ...

(a) $3 + f(x)$ (b) $4 + f(x)$ (c) 9 (d) 8.

(2) If $\lim_{x\to c}[f(x) \cdot g(x)] = 20$ and $f(x) = x + 1,\ g(x) = x$ then c = ...

(a) 2 (b) 3 (c) 4 (d) 5

(3) $\frac{d}{dx}(\sec x)$ = ...

(a) $\sec^2 x$ (b) $\sec^2 x \tan x$ (c) $\sec x \tan x$ (d) None of the above

(4) The integration is also known as

(a) summation (b) combination (c) antiderivative (d) None of the above

(5) $\int cosec^2 x\ dx$ = ...

(a) $\cot x$ (b) $\tan x$ (c) $-\cot x$ (d) $-\tan x$

(6) $\int -\frac{dx}{\sqrt{1-x^2}}$ = ...

(a) $\cos^{-1} x$ (b) $\sin^{-1} x$ (c) $\tan^{-1} x$ (d) None of the above

(7) If $\int f(x)\ dx = F(x)$ then $\int_a^b f(x)$ = ...

(a) $F(a) - F(b)$ (b) $F(b) - F(a)$ (c) 0 (d) None of the above

(8) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ = ...

(a)2π (b)π (c)$\pi/2$ (d)$\pi/4$

(9) Which of the following differential equations is of first order and first degree

(a) $x^2\frac{d^2y}{dx^2} = 1$ (b) $\frac{d^2y}{dx^2} + 2y = x^2$ (c) $\frac{dy}{dx} = 5y$ (d) None of the above

(10) $y - cx^2$ is solution of the differential equation

(a) $(y - x)dy + ydx = 0$ (b) $xy' = 2y$ (c) $xy' = x^2 + y$ (d) None of the above

Que.2 Answer the following (Any Ten) 20

(1) Evaluate $\lim_{x \to 2} \dfrac{x^3 - 8}{x^5 - 32}$

(2) Evaluate $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

(3) Find derivative of $\dfrac{\tan x}{x}$ with respect to x .

(4) Evaluate $\displaystyle\int \frac{x^3 + 3x^2 + 4}{\sqrt{x}}\, dx \quad , x > 0$

(5) Evaluate $\displaystyle\int \frac{1}{4x^2 + 9}\, dx$

(6) Evaluate $\int (e^{a \log x} + e^{x \log a})\, dx$

(7) State some useful results about definite integration .

(8) Evaluate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x \; dx$

(9) Evaluate $\displaystyle\int_0^1 \frac{2x + 3}{5x^2 + 1}\, dx$

(10) Obtain the order and degree of differential equation $x + \dfrac{dy}{dx} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^3}$.

(11) Obtain the differential equation of family of circles having centre on y - axis and radius 2 unit .

(12) Find the differential equation of $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b are arbitrary constants .

Que.3 (a) Evaluate $\lim_{x \to 2} \dfrac{x^3 - 3x^2 + 5x - 6}{x^3 - 8}$. 4

(b) By using definition find $\dfrac{d}{dx}(\sqrt{x})$. 3

(c) Find $\dfrac{dy}{dx}$ for $y = x^x + sinx^x$. 3

OR

Que.3 (a) Evaluate $\lim_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$. 4

(b) By using definition find $\dfrac{d}{dx}(x^4)$. 3

(c) Find $\dfrac{dy}{dx}$ for $x^3 + y^3 = \sin(x + y)$. 3

Que.4 (a) Evaluate $\int x\sqrt{x+2}\, dx, \quad x > -2$. 3

(b) Evaluate $\int x^2 \sqrt{a^6 - x^6}\, dx \quad (a > 0)$. 3

(c) Evaluate $\int x \log x \; dx$. 4

OR

Que.4 (a) Evaluate $\int \frac{e^{2x}+1}{e^{2x}-1}\,dx$. 3

(b) Evaluate $\int \frac{1}{\sqrt{2ax-x^2}}\,dx \quad (0<x<2a)$. 3

(c) Evaluate $\int x\cos x\,dx$. 4

Que.5 (a) Evaluate $\int_1^4 f(x)\,dx$, where $f(x)=\begin{cases}2x+8, & 1\le x\le 2\\ 6x, & 2<x\le 4\end{cases}$. 4

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,\cos^5\theta\,d\theta$. 3

(c) Evaluate $\int_0^1 \frac{dx}{\sqrt{x^2+4x+3}}$. 3

OR

Que.5 (a) Evaluate $\int_0^{\frac{1}{2}} \frac{sin^{-1}x}{(1-x^2)^{3/2}}\,dx$. 4

(b) If $\int_0^k \frac{\tan x}{1+\tan x}\,dx=\frac{\pi}{4}$, then find k . 3

(c) Evaluate $\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx$. 3

Que.6 (a) Solve $(1+x^2)dy=xydx$. 3

(b) Solve the differential equations $\frac{dy}{dx}=e^{x+y}$. Find the particular solution subject to initial condition , $y(1)=1$. Also find $y(-1)$. 4

(c) Verify that $y=(x+c)e^{-1}$ is the general solution of $\frac{dy}{dx}+y=e^{-x}$, where c is arbitrary constant . 3

OR

Que.6 (a) Solve $y(1+e^x)dy=(y+1)e^x dx$. 3

(b) Solve $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$. If $y(\pi/4)=\pi/4$, then find the particular solution of the given differential equation . 4

(c) Verify that $y=cx+\frac{1}{c}$ is the general solution of $y\left(\frac{dy}{dx}\right)=x\left(\frac{dy}{dx}\right)^2+1$, where c is arbitrary constant . 3

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