[7 / A-14] SC

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SARDAR PATEL UNIVERSITY

B.Sc. (SEM- II) Examination(Regular & NC)

Tuesday, 29th March-2016

USO2CMTHO2 : (MATRIX ALGEBRA AND DIFFERENTIAL EQUATIONS)

Time : 10:30 A.M. TO 12:30 P.M. Maximum Marks : 70

Note : Figures to the right indicate marks to the questions.

Q.1 Answer the following by selecting the correct choice from the given options. **[10]**

(1) A matrix A=$[a_{ij}]$ is said to be upper triangular matrix if , a_{ij}=0 for all ______.

(a) $i > j$ (b)$i < j$ (c)$i = 1$ (d) j=1

(2) Principal diagonal entries of skew- Symmetric matrix are all______.

(a) $zero$ (b) $comlex$ (c) $real$ (d) $none\ of\ these$

(3) Total number of elements in a matrix of 3 x 4 is ______.

(a) 4 (b) 7 (c) 12 (d) 3

(4) A square matrix A is said to be unitary matrix if ______ .

(a) $A.A^{-1} = 1$ (b) $A = A'$ (c) $A.A' = I$ (d) $A^{\theta}.A = I$

(5) If $|A + 4I| = 0$ then one of the characteristic root of A is______.

(a) -4 (b) 1 (c) 0 (d) 4

(6) Characteristic roots of a Hermitian matrix are______.

(a) $zero$ (b) $comlex$ (c) $real$ (d) $none\ of\ these$

(7) The complementary function of $(D^2 - 4D + 4)\ y = e^x + sin14x + x^5$ is______.

(a) $c_1e^{2x} + c_2e^{-2x}$ (b) $c_1e^{2x} - c_2e^{-2x}$ (c) $(c_1 + c_2x)e^{-2x}$ (d) $(c_1 + c_2x)e^{2x}$

(8) The Solution of $\frac{1}{D^2}e^x$=--------

(a) e^x (b) $2\,e^x$ (c) $\frac{1}{e^{2x}}$ (d) $\frac{1}{2!}e^x$

(9) $\frac{1}{f(D)}e^x\ sinx$=__________

(a) $e^x\ \frac{1}{f(D+1)}sinx$ (b) $-e^x\frac{1}{f(D-1)}sinx$ (c) $e^x\ \frac{1}{f(D-1)}sinx$ (d) $-e^x\frac{1}{f(D+1)}sinx$

(10) The Solution of $\frac{1}{D^2+9}$ cos3x =______

(a) $\frac{x}{6}$cos3x (b) $\frac{x}{6}$sin3x (c) $-\frac{x}{6}$cos3x (d) $-\frac{x}{6}$sin3x

Q.2 Attempt any Ten: **[20]**

(1) Define : Symmetric matrix with illustration.

(2) If A & B are two matrices of order $m \times n$ & $n \times p$ then P.T $(AB)^{\theta} = B^{\theta}A^{\theta}$.

(3) For A = $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ show that AA$^T = I$

(4) If A= $\begin{bmatrix} 1 & 3 & 5 \\ 2 & -4 & 9 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 4 & 3 \\ 3 & 9 & 8 \end{bmatrix}$ then find 2A+4B.

(5) Define : (1) Characteristic matrix (2) Characteristic polynomial

(6) Find the characteristic equation of A= $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(7) Let y_1 & y_2 be two solution of linear differential equation $\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + ---- + a_ny = 0$ & c_1, c_2 be two arbitrary constant .Then $c_1y_1 + c_2y_2$ is also a solution .

(8) Solve $(D^3 - 4D^2 + 5D - 2)\ y = 0$.

(9) Prove that $\frac{1}{D-\alpha}X = e^{\alpha x}\int X\ e^{-\alpha x}$ dx

(10) Find P.I. for $(D^2 + 9)$y= sin4x.

(11) Find the particular integral of $(D^3 + 4D)$y=$cos2x$

(12) Find the particular integral of $(D^3 + 1)$y=x^3

Q.3

(a) Show that every square matrix can be expressed in one &only one way as the sum of a symmetric & skew- symmetric matrix . [5]

(b) State and prove associative law for product of matrices. [5]

OR

Q.3

(a) Prove that every square matrix can be expressed in one and only one way as P+iQ ,where P & Q are Hermitian matrix. [5]

(b) If A=$\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that A^k= $\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ where k is positive integer. [5]

Q.4

(a) State and prove Cayley- Hamilton theorem. [5]

(b) Find the characteristic equation of the matrix A= $\begin{bmatrix} 0 & 1 & 2 \\ 3 & -3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$ and verify that it is satisfied by A. [5]

OR

Q.4

(a) If S is a real Skew-Symmetric matrix then prove that $I-S$ is non singular and the matrix $A=(I+S)(I-S)^{-1}$ is orthogonal. [5]

(b) Find the characteristic roots and any one of the characteristic vector of matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ [5]

Q.5

(a) Obtain rule for finding the particular integral of f(D)y=e^{mx} where m is constant. [5]

(b) Solve $(D^2+4)y = sec2x.$ [5]

OR

Q.5

(a) Solve $(D^2-5D+6)\ y = 4e^x$subject to the condition that y(0)=y'(0)=1.Hence find y(16). [5]

(b) Solve (D^3-1) y= $(e^x-1)^2$. [5]

Q.6 Solve : $x^3\ \frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 15(x - x^{-1})$ [10]

OR

Q.6 Solve (D^2-2D+1) y= $e^{3x}\ x^2$ [10]

X=X=X