[7 / A-16] SC

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SARDAR PATEL UNIVERSITY

B.Sc.SEM-II

28^{th} March 2016 , Monday

10:30 am to 12:30 pm

US02CMTH01

(Analytic Solid Geometry)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) .... is the set of all points of the space , which are equidistance from a fixed point .

(a) cone (b) ellipse (c) sphere (d) circle

(2) The surface $\frac{x^2}{9} - \frac{z^2}{4} = -5y$ represents a ....

(a) elliptic paraboloid (b) elliptic hyperboloid of two sheet
(c) hyperbolic paraboloid (d) elliptic hyperboloid of one sheet

(3) Cylinder has .... vertex.

(a) only one (b) finitely many (c) no (d) infinitely many

(4) Sphere $x^2 + y^2 + z^2 + 6x - 8y - 4z = 0$ is passing through the point ....

(a) $(-3, 4, 2)$ (b) $(-6, 8, 4)$ (c) $(0, 0, 1)$ (d) $(1, 2, 3)$

(5) Vertex of second degree homogeneous equation of cone is ....

(a) not possible (b) $(1, 2, 3)$ (c) (α, β, γ) (d) $(0, 0, 0)$

(6) Intersection of surface with xz - plane gives ....

(a) xy - trace (b) x- intercept (c) section by xy - plane (d) xz - trace

(7) The minor axis of the surface $-\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ is ....

(a) x (b) y (c) z (d) non of these

(8) Given fixed curve in the cone is called ....

(a) generator (b) circle (c) vertex (d) guiding curve

(9) Radius of sphere of $x^2 + y^2 + z^2 - 2x - 4y - 4z = 7$ is ....

(a) 7 (b) $\sqrt{7}$ (c) 4 (d) 49

(10) Reciprocal of reciprocal cone of given cone is ....

(a) sphere (b) given cone (c) line (d) cylinder

Q.2 Answer the following in short [Attempt any **ten**]: [20]

(1) Plot the points $(2, 7\pi/4, \pi/6)$.

(2) Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y - 6z + 5 = 0$.

(3) Identify the surface $4x^2 - 16y^2 + z^2 = 16$ also find its trace.

(4) Define Tangent line and Tangent plane to the cone. [PTO]

(5) Show that $Ax^2 + By^2 + Cz^2 = D$ represents an elliptic hyperboloid of one sheet if one coefficient is negative and $D > 0$.

(6) Define generator of cylinder and axis of cylinder.

(7) Prove that a sphere with centre (α, β, γ) and radius a is given by $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = a^2$.

(8) Find the equation of cone with vertex at the origin and which passes through the curve $ax^2 + by^2 = 2z$; $lx + my + nz = p$.

(9) Define reciprocal cone.

(10) Find the equations of the tangent plane and the normal line to the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z - 24 = 0$ at $(1, 1, 2)$.

(11) Define Cylinder and Right circular cylinder.

(12) Identify the surface $9x^2 + 144y^2 - 16z^2 = -144$.

Q.3

(a) Prove that the circles [5]
$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$; $5y + 6z + 1 = 0$ and
$x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$; $x + 2y - 7z = 0$.
lie on the same sphere and find its equation.

(b) Show that the plane lx+my+nz=p touches the sphere [5]
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ iff
$(l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d) = (ul + vm + wn + p)^2$

OR

Q.3

(c) Let two spheres be given by [5]
$S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$,
$S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$. Prove that
$S_1 + \lambda S_2 = 0$,where $\lambda \in \mathbb{R}$, $\lambda \neq -1$, represents a family of spheres passing through the intersection of the spheres $S_1 = 0$ *and* $S_2 = 0$.

(d) Find the equation of sphere which passes through (1,0,0),(0,1,0),(0,0,1) and has its radius as small as possible. [5]

Q.4

(a) By a proper choice of axes, Show that the Cartesian coordinates (x , y , z) of a point can be expressed in terms of spherical polar coordinates (ρ, θ, ϕ) as $x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. [5]

(b) Identify, describe and sketch the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. [5]

OR

Q.4

(c) Find jacobian of $x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. [5]

(d) Identify, describe and sketch the surface $\frac{x^2}{9} - \frac{y^2}{16} + \frac{z^2}{9} - 1 = 0$. [5]

Q.5

(a) Find a necessary and sufficient condition that the general equation of second degree may represent a cone. [5]

(b) Prove that the equation of the cone whose vertex is at origin is homogeneous and converse. [5]

OR

Q.5

(c) Find the equation of the tangent plane at point (α, β, γ) to the cone with vertex origin . [5]

(d) Find the equation of the cone whose vertex is at (α, β, γ) and whose generators intersects the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, z=0. [5]

Q.6

(a) Prove that the reciprocal cone of the reciprocal cone is the cone it self. [5]

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$. [5]

OR

Q.6

(c) Find the equation of the cylinder whose generators intersect the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0$ and are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$. [5]

(d) Find the equation of the right circular cylinder whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$, and whose radius is r . [5]

X=X=X