[30] No. of printed pages: 03

SARDAR PATEL UNIVERSITY
B. Sc. (Sem. IV) Examination
Monday, 15th April 2013
11.00 am – 1.00 pm
US04EMTH05 – Mathematics
(Calculus and Algebra - II)

Total Marks: 70

Note: Figures to the right indicate marks to the questions.

Q.1 Choose the most appropriate option for the following questions and write it down in the answer book. [10]

(1) f(a, b) is local minimum of f(x, y) if ________ condition satisfies.
(a) $AC-B^2 \le 0, A>0$ (b) $AC-B^2<0, A<0$
(c) $AC-B^2>0, A>0$ (d) $AC-B^2=0$

(2) If f(x, y) is sufficiently many times differentiable in some neighbourhood of (a, b) then $f_{xx}(a,b)=$ ________.
(a) A (b) B
(c) C (d) None

(3) For f(x, y) = x^2+y^2, $f_{xx}=$ __________.
(a) 2 (b) 2x
(c) 4x (d) 2y

(4) ∇f=________.
(a) $\sum \bar{i}\ \frac{\partial f}{\partial x}$ (b) $\sum \frac{\partial f}{\partial x}$
(c) $\sum \bar{i}\ x$ (d) None

(5) grad (8f) = __________.
(a) grad f (b) 8 grad f
(c) grad (8-f) (d) 0

(6) If f(x, y, z) = 3x+6y, then $\bar{\nabla} f$ = ___________.
(a) $3\bar{i}+3\bar{j}$ (b) $3\bar{i}+6\bar{j}$
(c) $6\bar{i}+3\bar{j}$ (d) $x\bar{i}+y\bar{j}$

(7) $\nabla\cdot(\nabla f)$ = __________.
(a) 0 (b) $2\nabla f$
(c) $\nabla^2 f$ (d) $(\nabla f)^2$

(8) For vector $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}, \nabla\cdot\bar{r}$ = __________.
(a) 3 (b) 0
(c) $\bar{i}+\bar{j}+\bar{k}$ (d) None

(9) If a, b $\in$ B, B is boolean algebra, then $a+a\cdot b$=___________.
(a) b (b) 0
(c) 1 (d) a

(10) If a$\in$B, B is boolean algebra, then $a\cdot a'$=__________.
(a) 1 (b) a
(c) 0 (d) a'

Q.2 Answer the following questions in short. **(Attempt Any Ten)** [20]

(1) Define: local maxima and local minima

(2) For f(x, y) = $x^2 + xy$, show that f(0, 0) is not an extreme value of f.

(3) Find A, B, C for the function f(x, y) = $x^3 + 3x + xy^2$

(4) Find unit normal vector to the surface x + y + z = 1 at (4, 2, -5)

(5) Show that $\nabla f(r) = f'(r)\nabla r$, where $\bar{r} = x\bar{i} + y\bar{j}$

(6) Find $\bar{\nabla}(\log r)$, where $\bar{r} = x\bar{i} + y\bar{j}$

(7) Define: Curl of a vector field

(8) In usual notations prove that $\bar{\nabla}\cdot(f\bar{v}) = f(\nabla\cdot\bar{v}) + \bar{v}\cdot\nabla f$

(9) In usual notations prove that, $\nabla\times(\nabla f) = 0$

(10) For a, b $\in$ B, the boolean algebra, prove $a\cdot(a+b) = a$

(11) Define: dual in boolean algebra and hence find dual of $x + y\cdot z = (x+y)\cdot(x+z)$

(12) Draw network represented by the function (x + y') (x' + y)

Q.3

(a) A rectangular box open at the top is have a volume of $32m^3$. Find the dimension of box so that the total surface area is minimum. [05]

(b) Show that $y^2 + x^2y + x^4$ has minimum at (0, 0) [05]

OR

Q.3

(a) Show that the function f(x, y) = $2x^4 - 3x^2y + y^2$ has neither maximum nor minimum at (0, 0). [05]

(b) Show that $x^3 - 3xy^2 + 2y^4$ has neither maximum nor minimum at (0, 0) [05]

Q.4

(a) Find directional derivative of f(x, y, z) = $2x^2 + 3y^2 + z^2$ at point (2, 1, 3) in the direction of $\bar{a} = \bar{i} - 2\bar{k}$ [05]

(b) Find gradient of the function $f(x, y) = \dfrac{x}{x^2 + y^2}$ at (2, 3) [05]

OR

Q.4

(a) Find ∇f for f(x, y, z) = $(x^2 + y^2 + z^2)^2$ at point (1, 2, 3) [05]

(b) Find unit normal vector to the surface $z^2 = 4(x^2 + y^2)$ at (1, 0, 2) [05]

Q.5

(a) Show that $\bar{\nabla}\cdot(r^n\bar{r}) = (n+3)r^n$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}|$ [05]

(b) Verify $\nabla\cdot(f\nabla g) = f\nabla^2 g + \nabla f\cdot\nabla g$ for f = x + y + z , g = xyz [05]

OR

Q.5

(a) Prove that $\bar{\nabla}\cdot(\bar{\nabla}\times\bar{v}) = 0$ [05]

(b) If f (x, y) = log $(x^2 + y^2)$, then prove that $\nabla\cdot(\nabla f) = \nabla^2 f = 0$ [05]

Q.6 State and prove De Morgan's laws for boolean algebra B. [10]

For a, b, c $\in$ B

If ab = ac and a + b = a + c , then prove that b = c.

OR

Q.6 Find boolean function for the following circuit. Simplify it and draw simplified circuit. [10]

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