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SARDAR PATEL UNIVERSITY
B. Sc. (Sem. IV) Examination
Saturday, 13th April 2013
11.00 am – 2.00 pm
US04CMTH02 – Mathematics (Differential Equations)

Total Marks: 70

Note: Figures to the right indicate marks to the questions.

Q.1 Choose the most appropriate option for the following questions and write it down in the answer book. [10]

(1) Integral curve of $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ is __________.

(a) $x=c_1 y; y=c_2 z$ (b) $x-y=c_1$; $x-z=c_2$

(c) $xy=c_1$; $yz=c_2$ (d) $yz+zx+xy=0$

(2) Orthogonal trajectories of given curves ___________.

(a) Intersect them at acute angle

(b) Intersect them at obtuse angle

(c) Intersect them at right angle

(d) are parallel to those curves

(3) Integral curve of $2dx=5dy=7dz$ is __________.

(a) $2x+5y+7z=c$ (b) $x^2=y^5,\ x^2=z^7$

(c) $2x-5y=c_1,\ 2x-7z=c_2$ (d) $2x+5y=c_1,\ 5y+7z=c_2$

(4) The general form of linear partial differential equation is _______.

(a) $Pp-Qq=R$ (b) $p+q=1$

(c) $pq=1$ (d) $Pp+Qq=R$

(5) ax-by+z=8 is a solution of ________.

(a) $pq-qy+z=8$ (b) $qx-py-z=8$

(c) $pq+qy-z=-8$ (d) $px+qy-z=-8$

(6) Eliminating the arbitrary constants from z=(x+a) (y+b) we get _____.

(a) $p+q=z$ (b) $p-q=z$

(c) $pq=z$ (d) $\frac{p}{q}=z$

(7) The surface orthogonal to one parameter family of surface f(x, y, z)=c are the surface generated by the integral curve of the equations _________.

(a) $\frac{dx}{\frac{\partial f}{\partial x}}=\frac{dy}{\frac{\partial f}{\partial y}}=\frac{dz}{\frac{\partial f}{\partial z}}$ (b) $\frac{dx}{\frac{\partial f}{\partial z}}=\frac{dy}{\frac{\partial f}{\partial x}}=\frac{dz}{\frac{\partial f}{\partial y}}$

(c) $\frac{dx}{\frac{\partial f}{\partial y}}=\frac{dy}{\frac{\partial f}{\partial z}}=\frac{dz}{\frac{\partial f}{\partial x}}$ (d) $\left(\frac{\partial f}{\partial x}\right)dx=\left(\frac{\partial f}{\partial y}\right)dy=\left(\frac{\partial f}{\partial z}\right)dz$

(8) Let $F(u,v)=0$, where u=y-x=c_1 and v=z-x=c_2 be the general solution of p+q=1 then the solution passing through the curve x=0, y^2=z is _________.

(a) $(y-x)^2=z$ (b) $(y-x)^2=z-x$

(c) $y-x=(z-x)^2$ (d) None

(9) Two differential equations f(x, y, z, p, q)=0 and g(x, y, z, p, q)=0 are compatible is ________.

(a) [f, g] = 0 (b) [f, g] = 1

(c) [f, g] = p (d) [f, g] = q

(10) For linear partial equation with constant coefficient F(D, D')z = f(x, y) the operator D'=_______.

(a) $\frac{\partial}{\partial x}$ (b) $\frac{\partial}{\partial p}$

(c) $\frac{\partial}{\partial q}$ (d) $\frac{\partial}{\partial y}$

Q.2 Answer the following questions in short. **(Attempt Any Ten)** [20]

(1) Find the integral curves of the equation xdx=ydy=zdz

(2) Solve: $\frac{dx}{2xz}=\frac{dy}{2yz}=\frac{dz}{z}$

(3) Solve: $\frac{dx}{1+x}=\frac{dy}{1+y}=\frac{dz}{z}$

(4) Verify whether the differential equation ydx+xdy-zdz=0 is integrable or not.

(5) Eliminate arbitrary function 'f' from the equation $z=xy+f(x^2+y^2)$

(6) Obtain partial differential equation of ax-by+4z=12

(7) Obtain partial differential equations of the form $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $5x^2+6y^2+7z^2=c$.

(8) Find integral surface of $x^2+y=c_1$, $xz+y=c_2$ passes through the line x=0, y=1.

(9) Find the differential equation of the surface which is orthogonal to $x^2+y^2+z^2 = cz$

(10) Find the general solution of the equation 2p+q=3

(11) Find the complete integral of pq=1

(12) If z=tx+yf(t)+g(t) then prove that $rt-s^2=0$

Q.3

(a) Solve: $\frac{dx}{x+z}=\frac{dy}{y}=\frac{dz}{z+y^2}$ [05]

(b) Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersection with the family of planes parallel to z=0. [05]

OR

Q.3

(a) Solve: $\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}$ [05]

(b) Solve: $\frac{dx}{x(y^2-z^2)}=\frac{dy}{y(z^2-x^2)}=\frac{dz}{z(x^2-y^2)}$ [05]

Q.4

(a) Integrate: 2xzdx+zdy-dz=0 [05]

(b) In usual notations prove that $p\frac{\partial(u,v)}{\partial(y,z)}+q\frac{\partial(u,v)}{\partial(z,x)}=\frac{\partial(u,v)}{\partial(x,y)}$ [05]

OR

Q.4

(a) Find the general solution of given linear partial differential equation $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$ [05]

(b) If X is a vector such that $X\cdot curl\,X=0$ and μ is an arbitrary function of x, y, z then prove that $(\mu X)\cdot curl(\mu X)=0$ [05]

Q.5

(a) Find the integral surface of the equation $(x-y)\,y^2p+(y-x)x^2q=(x^2+y^2)z$ which passes through $xz=a^3,\ y=0$. [05]

(b) Find the surface which intersects the surfaces of the system z(x+y) = c(3z+1) orthogonally and which passes through the circle $x^2+y^2=1$, z=1. [05]

OR

Q.5

(a) Show that $(x-a)^2+(y-b)^2+z^2=1$ is the complete integral of non-linear p.d.e. $z^2(1+p^2+q^2)=1$. Determine a general solution by finding the envelope of its particular solution. [05]

(b) Find the integral surface of the linear p.d.e. $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line x+y=0, z=1. [05]

Q.6 Prove that the equation $f(x,y,p,q)=0$ and $g(x,y,p,q)=0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)}+\frac{\partial(f,g)}{\partial(y,q)}=0$ and verify that the equations $p=P(x,y)$ and $q=Q(x,y)$ are compatible if $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$. Also find complete integral of p+q=pq. [10]

OR

Q.6 If $\mu_1,\mu_2....\,\mu_n$ are solutions of homogeneous linear partial differential equation $F(D,D')z=0$ then prove that $\sum_{r=1}^{n}c_r\mu_r$ is also solution of $F(D,D')z=0$ where c_r are arbitrary constants. Hence solve $\frac{\partial^4 z}{\partial x^4}+\frac{\partial^4 z}{\partial y^4}+2\frac{\partial^4 z}{\partial x^2\partial y^2}$. [10]

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