[A-8] SC

SEAT No.________ No of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - IV)(2010-Batch) EXAMINATION - 2019
Saturday , 13^{th} April , 2019
MATHEMATICS : US04EMTH05
(Calculus and Algebra - 2)

Time : 10:00 a.m. to 12:00 noon Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) If $f(x,y) = x^4 - 2x^2 - 2y^2 + 4xy + y^4$ then stationary points are ....

(a) (0,0) (b) $(0,0)$, $(\sqrt{2}, -\sqrt{2})$ (c) $(\sqrt{2}, \sqrt{2})$ (d) $(0,0)$, $(\sqrt{2}, -\sqrt{2})$, $(-\sqrt{2}, \sqrt{2})$

(2) If $f(x,y) = x^3 + y^3 - 63x - 63y + 12xy$ then f_{yy}=.......

(a) 6y (b) 63y (c) $3y^2$ (d) None

(3) Let $E \subset R^2$ and $F : E \to R$ admits the first order partial derivative at $(a,b) \in E$ then $f_x(a,b) = f_y(a,b)$=0 if f =........

(a) has local extremum at (a,b) (b) is constant (c) zero function(d)differential function

(4) A unit normal vector to the surface $f(x,y,z) = 0$ is defined as $\overline{n}$ = ...

(a) $\overline{\nabla} f$ (b) $|\overline{\nabla} f|$ (c) $\dfrac{\overline{\nabla} f}{|\overline{\nabla} f|}$ (d) None

(5) ...... is Laplace's equation.

(a) $\nabla^2 f \geq 0$ (b) $\nabla^2 f \leq 0$ (c) $\nabla^2 f = 0$ (d) $\nabla^2 f = \nabla f$

(6) $\overline{\nabla} f$ = ...

(a) $\Sigma \bar{i} \frac{\partial f}{\partial x}$ (b) $\Sigma i \frac{\partial f}{\partial x}$ (c) Σix (d) None

(7) $\overline{\nabla} \cdot (f \overline{\nabla} g)$=......

(a) $f\overline{\nabla}^2 g + \overline{\nabla} f \cdot \overline{\nabla} g$ (b)$g\overline{\nabla}^2 f - \overline{\nabla} f \cdot \overline{\nabla} g$ (c)$f\overline{\nabla}^2 g - \overline{\nabla} f \cdot \overline{\nabla} g$ (d) None

(8) If $\overline{r} = x\overline{i} + y\overline{j}$ then $\dfrac{\partial \overline{r}}{\partial x}$ = ...

(a) $\overline{i}$ (b) $\dfrac{x}{r}$ (c) 1 (d) $\overline{i} + y\overline{j}$

(9) In Boolean algebra $a \cdot a$ = ...

(a) 1 (b) 0 (c) a (d) $2a$

(10) $a + a'$ = ...

(a) a (b) a' (c) 1 (d) 0

Que.2 Answer the following (Any Ten) 20

(1) Find stationary points of $x^2 + y^2 + 6x + 12$.

(2) Find stationary points of $x^2 + 2xy + 2y^2 + 2x + y$.

(3) Define Extreme point , Stationary point , Saddle point .

(P.T.O.)

(4) Prove that $\overline{\nabla}(f-g)=\overline{\nabla}f-\overline{\nabla}g$.

(5) Prove that $\overline{\nabla}(fg)=f\overline{\nabla}g+g\overline{\nabla}f$.

(6) Prove that $\overline{\nabla}\left(\frac{f}{g}\right)=\frac{g\overline{\nabla}f-f\overline{\nabla}g}{g^2}$.

(7) If $f(x,y)=log(x^2+y^2)$ then prove that $\overline{\nabla}^2 f=0$.

(8) Find gradient of function $f(x,y)=\frac{y}{x^2+y^2}$ at (2 , 3) .

(9) Prove that $\overline{\nabla}(f+g)=\overline{\nabla}f+\overline{\nabla}g$.

(10) For every element a & b in Boolean algebra B,prove that (i) $a+1=1$ (ii) $a\cdot 0=0$.

(11) Define Boolean Algebra and state its Properties.

(12) Simplify the function $x+xy'$ and then draw the network represented by them .

Que.3 (a) Show that $y^2+x^2y+x^4$ has a minimum at $(0,0)$. 5

(b) Show that $(y-x)^4+(x-2)^4$ has minimum at $(2,2)$. 5

OR

Que.3 (c) Show that $2x^4-3x^2y+y^2$ has neither a maximum nor a minimum at $(0,0)$. 5

(d) A rectangular box open at the top is to have a volume of $32m^3$. Find the dimension of box so that the total surface area is minimum . 5

Que.4 (a) $\overline{\nabla}f(r)=f'(r)\overline{\nabla}r=f'(r)\frac{\overline{r}}{r}$, where $\overline{r}=x\overline{i}+y\overline{j}+z\overline{k}$, $r=|\overline{r}|$ 5

(b) $\overline{\nabla}\left(\frac{1}{r}\right)=-\frac{\overline{r}}{r^3}$, where $\overline{r}=x\overline{i}+y\overline{j}+z\overline{k}$,$r=|\overline{r}|$ 5

OR

Que.4 (c) Find direction derivative of $f(x,y,z)=2x^2+3y^2+z^2$ at point (2,1,3) in the direction of $\bar{a}=\overline{i}-2\overline{k}$. 5

(d) Find unit normal vector of surface $z^2=x^2+y^2$ at (3 , 4 , 5) . 5

Que.5 (a) Prove that $\overline{\nabla}\cdot(f\overline{\nabla}g)=f\overline{\nabla}^2 g+\overline{\nabla}f\cdot\overline{\nabla}g$. Hence prove that $\overline{\nabla}\cdot(f\overline{\nabla}g-g\overline{\nabla}f)=f\overline{\nabla}^2 g-g\overline{\nabla}^2 f$. 5

(b) Prove that $\overline{\nabla}\cdot(\bar{u}\times\bar{v})=\bar{v}\cdot(\overline{\nabla}\times\bar{u})-\bar{u}\cdot(\overline{\nabla}\times\bar{v})$ 5

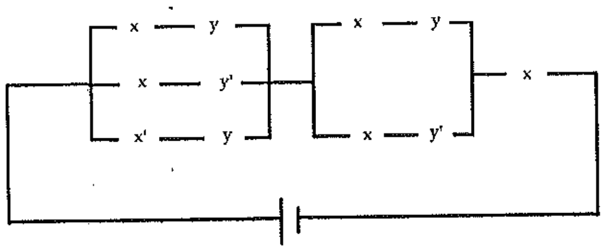
OR

Que.5 (c) Prove that $\overline{\nabla}\times(f\bar{v})=f(\overline{\nabla}\times\bar{v})+\overline{\nabla}f\times\bar{v}$. 5

(d) Verify $\overline{\nabla}\cdot(f\bar{v})=f(\overline{\nabla}\cdot\bar{v})+\bar{v}\cdot\overline{\nabla}f$ for $f=e^{2xyz}$ and $\bar{v}=x\overline{i}+2y\overline{j}+3z\overline{k}$. 5

Que.6 (a) If a and b are elements of boolean algebra B,satisfying the relation $a \leqslant b$ then prove that $a + bc = b(a + c)$, $\forall c \in B$. 5

(b) Find the boolean function of switching circuit given here and then it's equivalent simplified circuit, also draw the simplified circuit. 5



OR

Que.6 (c) Prove that for every a & $b \in B$, $(ab)' = a' + b'$ & $(a + b)' = a'b'$. 5

(d) Find the boolean function of switching circuit given here and then it's equivalent simplified circuit, also draw the simplified circuit. 5