SEAT No.________

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[2/A-5]

SARDAR PATEL UNIVERSITY

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B.Sc. Sem-IV

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10.00 a.m. to 12.00 noon

Sub: MATHEMATICS (US04EMTH05)

Calculus and Algebra-2

Maximum Marks: 70

Q.1 Write the correct answer from given alternative. [10]

(1) In Boolean algebra ,$1' =$_________

(a) 0 (b) 1 (c) a (d) None

(2) If $f(x,y) \geq f(a,b)$, for all $(x,y) \in E \subset \mathbb{R}^2$, then f is said to have.......

(a) local extreme point at (a,b)

(b) global minimum point at (a,b)

(c) global maximum point at (a,b)

(d)constant value at (a,b)

(3) Let $E \subset \mathbb{R}^2$ and $f : E \to \mathbb{R}$ admits the first order partial derivatives at $(a,b) \in E$ and $f_x(a,b) = f_y(a,b) = 0$. Define $A = f_{xx}(a,b)$, $B = f_{x,y}(a,b)$ and $C = f_{yy}(a,b)$. Then $f(a,b)$ is a local maximum of f if ....

(a) $AC - B^2 > 0$ and $A > 0$ (b) $AC - B^2 > 0$ and $A < 0$ (c) $AC - B^2 < 0$ (d) $AC - B^2 = 0$

(4) $\overline{\nabla} f =$ __________.

(a) $\Sigma\, \bar{i} \cdot \dfrac{\partial f}{\partial x}$ (b) $\Sigma\, \bar{i} \dfrac{\partial f}{\partial x}$ (c) $\Sigma\, \bar{i}x$ (d) None

(5) If $\nabla \cdot V = 0$, then V is called.......

(a) solenoidal (b) irrotational (c) harmonic (d) Laplacian

(6) The divergent of a vector field $V = x^3 + y$ is........

(a) $3x^3i + yj$ (b) $3x^2i + j$ (c) $3x^3i + j$ (d) $3x^2i + yj$

(7) $\nabla^2 = \Sigma \dfrac{\partial^2}{\partial x^2}$ is called __________.

(a) Laplacian Operator (b) Gradient of scalar field

(c) Vector differential Operator (d) None

(8) grad(7f)=......

(a) grad f (b) 7 grad f (c) grad 7-f (d) 0

(9) The primary law, for $a \in \mathbb{B}$, implies $a \cdot a' =$ ......

(a) 1 (b) a (c) 0 (d) a'

(10) $a = a + a \cdot b$ is called law of........

(a) Distributive (b) Absorption (c) Associative (d) Commutative

Q.2 Answer any ten in short. [20]

(1) Define Stationary point and global minima.

(2) State principle of duality.

(3) State a sufficient condition for the existence of extreme values of a function.

(4) Show that $\nabla \cdot (\alpha V) = \alpha(\nabla \cdot V)$, where α be any scalar.

(5) Show that $\nabla \times (\nabla f) = 0$.

(6) Define Directional derivative and Normal vector.

(7) For every element a and b in boolean algebra B prove that $a + a = a$.

(8) For every boolean algebra B, prove that $(a')' = a$.

(P.T.O.)

(9) Find stationary points of $f(x,y) = x^2 + y^2 + 6x + 12$.

(10) Show that $\nabla f(r) = f'(r)\nabla r$

(11) Define divergent of a vector field,curl of vector field.

(12) Verify $\nabla \cdot (V_1 + V_2) = \nabla \cdot V_1 + \nabla \cdot V_2$ for $V_1 = x^2 + y$ and $V_2 = e^x$.

Q.3(a) A rectangular box open at the top is to have a volume of $32m^3$. Find the dimension of box so that the total surface area is minimum. [6]

(b) Show that $2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at $(0,0)$. [4]

OR

Q.3(c) Investigate the maxima and minima of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. [6]

(d) Show that $y^2 + x^2y + x^4$ has a minimum at $(2,2)$. [4]

Q.4(a) Prove $\nabla\frac{1}{r} = -\frac{\bar{r}}{r^3}$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}, r = |\bar{r}|$ [5]

(b) Prove $\nabla(f \pm g) = \nabla f \pm \nabla g$ [5]

OR

Q.4(c) Find directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at point $(2,1,3)$ in the direction of $\bar{a} = \bar{i} - 2\bar{k}$. [5]

(d) Find unit normal vector at the surface $z^2 = 4(x^2 + y^2)$ at $(1,0,2)$. [5]

Q.5(a) Find *curl* V for $V = \bar{r}|\bar{r}|^{-3}$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}, r = |\bar{r}|$. [5]

(b) For a differentiable scalar field f and differentiable vector field V, prove that $\nabla \times (fV) = f(\nabla \times V) + \nabla f \times V$. [5]

OR

Q.5(c) Show that $\nabla \cdot (r^n\bar{r}) = (n+3)r^n$, where $x\bar{i} + y\bar{j} + z\bar{k},\ r = |\bar{r}|$ [5]

(d) Verify $\nabla \cdot (f \nabla g) = f \nabla^2 g + \nabla f \nabla g$ for $f = x + y + z, g = xyz$. [5]

Q.6(a) State and prove De-Morgan's laws for Boolean algebra. [6]

(b) Draw the network represented by Boolean function $z(x' + yz + z')$. [4]

OR

Q.6(c) For every element a,b and c in boolean algebra B for which both the condition $ab = ac$ and $a + b = a + c$ holds, then prove that $b = c$. [6]

(d) Find the Boolean function of switching circuit given below and then its equivalent circuit, also draw the simplified circuit. [4]