SEAT No.________ No. of Printed Pages : 3

[1/A-

Sardar Patel University

B.Sc. Semester-IV Examination

Tuesday, 30th April, 2019

Subject:-PROBABILITY DISTRIBUTIONS

Paper Code:- US04CSTA02

Time:- (10:00 A.M. to 1:00 P.M.) M.Marks:7

Note:- (i) Simple/ Scientific calculator is allowed. (ii) Graph paper will be provided on request.
(iii) Statistical Table is allowed/ provided on request.

Q.1. **Multiple Choice Questions:-** [10]

(1) The mean and variance of a binomial distribution are 8 and 4 respectively. Then, P(X = 1) is equal to ________.
(a)0.0030 (b) 0.0300 (c)0.0003 (d)0.3000

(2) If $f(x) = \frac{1}{21}$, x = 1,2,......21.
= 0 , otherwise; is the p.m.f. of X then P(3 < X < 9) =________.
(a) $\frac{5}{21}$ (b) $\frac{8}{21}$ (c) $\frac{2}{21}$ (d) none

(3) If $f(x) = \binom{10}{x} p^x q^{10-x}$, x = 0,1,2,........, 10 and zero otherwise then $M_X(t)$ = ____.
(a) $(q + pe^t)^{10}$ (b) $(p + qe^t)^{10}$ (c) $(p + qt)^{10}$ (d) $(q + pt)^{10}$

(4) If $f(x) = kx^2(1-x)$, $0 \le x \le 1$.
= 0 , otherwise ; is the p.d.f. of X then k = ________.
(a) 8 (b) 10 (c) 12 (d) 14

(5) The value of P (-2.25 < Z< 1.25) using the standard normal distribution is______.
(a) 0.8786 (b) 0.4878 (c) 0.8822 (d) 0.8944

(6) If X_1 and X_2 are two independent random variables with m.g.f. respectively $M_{x_1}(t)$ and $M_{x_2}(t)$ and if Y = $X_1 + X_2$, then $M_Y(t)$ = __________.
(a)$M_{x_1}(t)+ M_{x_2}(t)$ (b)$M_{x_1}(t)-M_{x_2}(t)$ (c) $M_{x_1}(t).M_{x_2}(t)$ (d) none

(7) If X_1 and X_2 are two independent exponential variate with mean θ each, then Y = $X_1 + X_2$ ~ --------- distribution.
(a) G(2,θ) (b) G(2,2θ) (c) G(1,θ) (d) none

(8) If $X_i \sim NID(\mu, \sigma^2)$ distribution for i = 1,2,...... n then $\bar{X}$ ~ ______distribution
(a) $N(\mu, n\sigma^2)$ (b) $N(\mu, \frac{\sigma^2}{n})$ (c) $N(\mu, \sigma^2)$ (d) none

(9) If X and Y are independent and if X ~ N(0 , 1) and Y ~ $\chi^2_{(r)}$ distribution then $\frac{X}{\sqrt{Y/r}}$ ~ _____ distribution.
(a) $\chi^2_{(r)}$ (b) $t_{(r)}$ (c) $F_{(r,1)}$ (d) none

(10) If X ~ $F_{(r_1, r_2)}$ then $\frac{1}{X}$ ~ _______ distribution.
(a) $F_{(r_2, r_1)}$ (b) $F_{(r_1 * r_2)}$ (c) $F_{(r_1/r_2)}$ (d) none

Q.2. **Short Type Questions:- (Attempt Any Ten)** [20]

(1) If X has a Poisson variate such that P(X = 2) = 9P(X = 4) + 90 P(X = 6). Find (i) m (ii) the mean and variance of X.

(2) If $M_X(t) = (\frac{1}{3} + \frac{2}{3}e^t)^9$ is the m.g.f. of the random variable X, then
(i) identify the distribution of the random variable X. (ii) State its mean, variance.

(3) Obtain recurrence relation for the Binomial distribution.

(4) Define an Exponential distribution. Obtain it as a particular case of Gamma distribution.

(5) If $f(x) = \frac{k}{12}$, 0 < x < 12 and zero otherwise then find k and c.d.f. of X. Name the

[P.T.O.]

(6) If X ~ N (30 , 25) distribution then find (i) $P(26 \leq X \leq 40)$ (ii) $P(|X - 30| > 5)$.

(7) If $M_x(t) = (1 - 2t)^{-3}$, indentify the distribution of X and write the p.d.f. of x . State its mean and variance.

(8) If X ~ b(100, 0.5) then find P(X = 55). State the result you used

(9) If X and Y follows respectively b(5, 0.3) and b(3,0.3) distribution. What is the distribution of Z = X + Y.

(10) Define F – distribution. Write the p.d.f. of $F_{(2,2)}$ distribution.

(11) If $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ then show that $E(\bar{X}) = \mu$.

(12) Define chi-square distribution. State its relation with (i) Normal distribution (ii) Gamma distribution.

Q.3. (a) Define Hyper geometric distribution and derive the distribution. State its mean, variance and m.g.f. [05]

(b) If $M_X(t) = e^{3(e^t - 1)}$, is the m.g.f. of a random variable X, then find c.g.f. of X and hence find β_1 and β_2 . [05]

OR

Q.3. (a) Prove in usual notations for Binomial distribution b(n,p) that [05]

$\mu_{r+1} = pq\,(nr\,\mu_{r-1} + \frac{d\,\mu_r}{dp})$.

(b) If a random variable X follows a negative binomial distribution with mean = 12 and variance = 36 then find the p.m.f. of X and also the P(X = 0) and $P(X \geq 1)$. [05]

Q.4. (a) Show that for a Normal distribution, all the odd central moments are zero and even order central moments are given by [05]

$\mu_{2r} = \frac{\sigma^{2r}\,(2r)!}{2^r\, r!}$, r = 1,2,..........

(b) Let X has an exponential distribution with mean 40. [05]

Find (i) $E(X^r)$ (ii) P (X < 22) (iii) $P (X \geq 25)$ (iv) P (X > 22/ X > 20).

OR

Q.4. (a) Define beta distribution of first kind. Obtain its mean , variance and harmonic mean. [06]

(b) The monthly incomes of a group of 10,000 persons were found to be normal distributed with Mean Rs. 750and standard deviation 50. [04]

(i) What percentage of persons having income between Rs. 550 to Rs. 680?

(ii) What was the lowest income of among the richest 100?

Q.5. (a) If $X_1, X_2, \ldots X_r$ are independent random variables with m.g.f.s [05]

$M(t) = \frac{2}{3}(1 - \frac{1}{3}e^t)^{-1}$ then find m.g.f. of $Y = \sum_1^r X_i$. Name the distribution of Y.

(b) If $X_1, X_2, \ldots X_n$ denote a random sample of size n from normal distribution with mean μ and variance σ^2 then find by using m.g.f. the distribution [05]

(i) $Y = \sum_1^n X_i$ (ii) $\bar{X} = \frac{1}{n}\sum_1^n X_i$.

OR

Q.5. (a) If X_1, X2 follows binomial distribution with parameters (n_1; p) and (n_2; p) respectively, then prove that $Y = X_1 + X_2$ follows b($n_1 + n_2$; p).State the result you used. [05]

(b) If X_1, X_2 are independent random variables with N (5,25) and N(3,25) then state the distribution of $Y = X_1 + X_2$. Also state its p.d.f. mean and variance of Y. Also find P(Y< 21) and P (|Y|> 21). [05]

Q.6. (a) If $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ and $S^2 = \frac{1}{n}\sum_1^n(X_i - \bar{X})^2$ and $S'^2 = \frac{1}{n-1}\sum_1^n(X_i - \bar{X})^2$. Obtain $E(S^2)$ and $E(S'^2)$. [05]

(b) Let $\bar{x}$ and S^2 be the sample mean and variance associated with a r.s. of size n =16 from the normal distribution N (μ , 225). [05]

(i) Find constant 'a' and 'b' so that $P (a \leq S^2 \leq b) = 0.95$

(ii) Find constant 'c' so that P (- c ≤ $\frac{(\bar{x}-\mu)}{s}$ ≤ c)= 0.95

OR

Q.6. (a) If X is $\chi^2_{(23)}$, find (i) P (14.85 < X < 32.01) (ii) constant 'a' and 'b' such that P (a < X < b) =0.9 and P (X < a) = 0.05. [05]

(b) (i)If S_1 and S_2 are the standard deviations of independent r.s. of size $n_1 = 61$ and $n_2 = 31$ from the normal populations with $\sigma_1^2 = 12$ and $\sigma_2^2 = 18$. Find P ($\frac{S_1^2}{S_2^2}$ > 1.16). [05]

(ii)If S_1^2 and S_2^2 are the Variance of independent r.s. of size $n_1 = 10$ and $n_2 = 15$ from the normal populations with equal variances. Find P ($\frac{S_1^2}{S_2^2}$ < 4.03).

——— x ———