[A-15] SEAT No.________

No. of Printed Pages : 02

Sardar Patel University

B.Sc. Semester – IV Examination (NC - 2010 Batch)

Friday, 12th April, 2019

Course code: US04CSTA02

(Probability Distributions)

Time: 10.00 to 1.00 P.M

Maximum Marks: 70

Note: (i) Statistical table is allowed/provided on request (ii) Q.3 to Q.8 each sub question has 4 marks.

Q.1 Multiple Choice Questions (10×1)

(1) Poisson distribution is a limiting case of

(a) Binomial distribution (b) Negative Binomial distribution (c) Hypergeometric distribution (d) Both (a) and (b)

(2) The m.g.f of a r.v. X is $M_X(t) = (1-2t)^{-10}$ then $P(X < 16.27) =$ ________

(a) 0.3 (b) 0.7 (c) 0.1 (d) 0.9

(3) Let $X \sim N(0,1)$, what is the value of C so that $P(|X| \leq C) = 0.99$?

(a) 2.58 (b) 2.33 (c) 1.645 (d) None of the above

(4) Let X be a r.v. which is distributed uniformly between 0 and 2. What are the mean and variance of X?

(a) $1, 1/3$ (b) $0, 1$ (c) $1, 1/\sqrt{3}$ (d) $1, 1$

(5) If $X_i, i = 1, 2, \ldots n$ be n independent standard normal variates then $\sum_{i=1}^{n} Xi$ has

(a) $N(0,1)$ (b) $N(0,n)$ (c) χ_1^2 (d) χ_n^2

(6) Let the r.v. X have moment generating function $M_X(t) = e^{2t(1+t)}$ then $P(X \leq 2) =$ ________

(a) 1/4 (b) 1/2 (c) 1/3 (d) 2/3

(7) If $f(x) = \frac{K}{20}$, $1 < x < 21$ *and zero otherwise*, is the pdf of X then $V(X) =$ ________

(a) 1.11 (b) 11 (c) 21.21 (d) 33.33

(8) Geometric distribution is a particular case of ________ distribution.

(a) Binomial (b) Poisson (c) Negative Binomial (d) Hypergeometric

(9) Let X and Y be two independent standard normal variates, the value of $P(X - Y < 1)$ equals

(a) 0 (b) 0.5 (c) 1 (d) $P(X + Y < 1)$

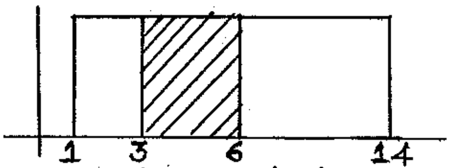
(10) If the random variable Z is the standard normal score, which of the following probabilities could easily be determined without referring to a table?

(a) $P(Z < 0)$ (b) $P(Z < -1.82)$ (c) $P(Z > 1.02)$ (d) $P(Z < -2.01)$

Q.2 Short Type Questions (Attempt Any Six) (6×2)

(a) Obtain recurrence relation for the probabilities of Geometric distribution. State its uses, if any.

(b) The following graph shows the uniform distribution of waiting times, in minutes, at Anand railway station. Find the area of the shaded region.



(c) Define Hypergeometric distribution. State its mean and variance.

(d) If $M_X(t) = (1-2t)^{-4}$, is the m.g.f of a r.v. X then find c.g.f and hence find β_1, β_2.

(e) Let X be a normal variate with mean 200 and $P(X > 225) = 0.1587$, Determine $P(X < 175)$.

(f) If $f(x) = \frac{1}{2a}, -a < x < a$ *and zero otherwise*, is the p.d.f. of X. Show that $M_X(t) = \frac{1}{(at)} Sinh(at)$.

(g) Examine whether $f(x) = (1-p)^{x-1}p,\ x = 1, 2, 3, \ldots$, is p.m.f or not?

(h) State and prove additive property of Poisson distribution.

Q.3(a) Define Poisson distribution. Give two real life examples where Poisson distribution may be used. Obtain its mean and variance.

(b) The probability mass function of a r.v. X is

$P(X = x) = \left(\frac{2}{3}\right)^x \left(\frac{1}{2}\right), x = 1, 2, 3, \ldots$ and zero otherwise, Find $V(X)$ and $P(X > 2)$

OR

Q.3(a) Define Negative Binomial distribution. Obtain its mean and variance.

(b) Products produced by a machine has 20% defective rate.

(P.T.O)

(i) What is the prob. that the first defective occurs in the fifth item inspected? (ii) What is the average number of inspections to obtain the first defective?

Q.4(a) Define Binomial distribution. State the conditions under which Binomial distribution tends to Poisson distribution. Derive it.

(b) Let X and Y be two independent binomial variates with parameters $(2,p)$ and $(4,p), 0 < p < 1$. If $P(X \geq 1) = 5/9$, determine $P(Y \geq 1)$, $P(X + Y \geq 3)$.

OR

Q.4(a) If X denotes the no. of failures preceding the first success, in an infinite series of independent trials with constant prob. p of success for each trial. Identify the distribution of X and obtain the m.g.f of X and hence mean and variance of X.

(b) A life insurance agent sells on the average 3 life insurance policies per week. Use Poisson distribution to calculate the probability that in a given week he sells (i) some policies (ii) 2 or more policies.

Q.5(a) Do as directed:

(i) If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X > 0)$.

(ii) Define Beta distribution of first kind. Obtain its mean.

(b) The length X of component produced by a machine with pdf

$f(x) = k(1-x), 0 < x < 1$ *and zero otherwise*, where k is constant.

(i) Find the value of k (ii) Determine the mean of X (iii) Show that the prob. that the length of a randomly chosen component will be less than the mean is $5/9$. (iv) If 20 components are produced by independent operations of the machine, calculate the prob. that at least 2 of the components will have length less than mean.

OR

Q.5(a) Define Normal distribution. Obtain its m.g.f and hence show that $\beta_1 = 0$ and $\beta_2 = 3$.

(b) The lengths of certain species of fish are normally distributed with mean 60 cm. If 80% of fish are of length more than 42 cm; determine the value of standard deviation.

Q.6(a) Define Exponential distribution. Obtain its m.g.f and hence mean and variance.

(b) The length X of a component produced by a machine has probability density function given by

$f(x) = kx(1-x)^2, 0 < x < 1$ *and zero otherwise, where k is constant*

(i) Find the value of k (ii) Determine the mean of X (iii) find the prob. that the length of a randomly chosen component will be less than the mean. (iv) If 12 components are produced by independent operations of the machine, calculate the prob. that at least 2 of the components will have length less than mean.

OR

Q.6(a) Let $X \sim U(-a, a)$. Show that all the odd order moments are zero. Obtain an expression for even order moments.

(b) A die is rolled independently 120 times. Approximate the probability that

(i) More than 42 rolls are odd numbers (ii) the number of two's and three's is from 40 to 45 times.

Q.7(a) Let $Xi \sim N(\mu i, \sigma i^2), i = 1, 2 \ldots n$ be n independent variates. Obtain the distribution of $\sum Xi$. Identify and name it.

(b) Let $Z_1, Z_2, \ldots, Z_{10}$ be a r.s .of size $n = 10$ from the standard normal distribution $N(0,1)$. Let $W = Z_1^2 + Z_2^2 + \cdots + Z_{10}^2$. What will be the distribution of W? Derive it. Find the constant k so that $P(W < k) = 0.95$.

OR

Q.7(a) Show that the difference of two independent normal variates is also a normal variate.

(b) The m.g.f of a r.v. X is $M(t) = (0.4 + 0.6e^t)^{32}$. Find the approximate value of $(i) P(2 < X \leq 7)$ $(ii) P(X > 9)$. State clearly, the result you have used to solve the required probability.

Q.8(a) Let $X_i, i = 1, 2, \ldots n$ be a r.s. of size n from $N(0,1)$ distribution.

Define $\bar{X}_m = \frac{1}{m}\sum_{i=1}^{m} Xi$ and $\bar{X}_{n-m} = \frac{1}{n-m}\sum_{i=m+1}^{n} Xi$.

What will be the distribution of (i) $\frac{1}{2}(\bar{X}_m + \bar{X}_{n-m})$ (ii) $m\bar{X}_m^{\,2} + (n-m)\,\bar{X}_{n-m}^2$

(b) Let X has t - distribution with 14 d.f. Determine the value of K such that

$(i) P(|X| > k) = 0.01$ (ii) $P(X^2 > K) = 0.99$

OR

Q.8(a) Suppose that the weights in lbs of American adult can be represented by a normal variate with mean 150 lbs and variance 900 lb^2. An elevator containing a sign "Maximum 12 people can safely carry 2000 lbs". Find the probability that 12 people will not overload elevator. State and prove the result you have used to calculate the required probability.

(b) Let $\bar{X}$ and $\bar{Y}$ be the means of samples of sizes $n_1 = 4$ and $n_2 = 9$ from two normal populations with means $\mu_1 = 2, \mu_2 = 4$ and variances $\sigma_1^2 = 6$ and $\sigma_2^2 = k$. If $P(\bar{X} - \bar{Y} > 8) = 0.0228$, then what is the value of 'k'?

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(2)