[2/A-1] SEAT No.________ No. of Printed Pages : 3

Sardar Patel University, Vallabh Vidyanagar

B.Sc. [Semester-IV] Examinations : 2019

Subject : Mathematics US04CMTH02 Max. Marks : 70

Differential Equations

Date: 27/04/2019, Saturday Timing: 10.00 am - 01.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] Integral curve of $ax^2dx = by^2dy = cz^2dz$ is given by

[A] $ax^3 - by^3 = c_1, by^3 - cz^3 = c_2$ [B] $ax^3 + by^3 = c_1, by^3 + cz^3 = c_2$

[C] $ax^3 + by^3 = c_1, by^3 - cz^3 = c_2$ [D] $ax^3 - by^3 = c_1, by^3 + cz^3 = c_2$

[2] Solution of $udx = vdy = wdz$ is given by

[A] $ux - vy = z$; $vy - wz = x$ [B] $ux - vy = c_1$; $vy - wz = c_2$

[C] $ux + vy = z$ [D] vy-wz=x

[3] The equations $x - y = c_1; y + z = c_2$, represent integral curve of

[A] $dx = dy = -dz$ [B] $dx = dy = dz$ [C] $dx = -dy = dz$ [D] $-dx = dy = dz$

[4] The differential equation $p + q = 0$ is

[A] linear partial differential equation

[B] non-linear partial differential equation

[C] Paffian differential equation

[D] none of these

[5] The general solution of the partial differential equation $px + qy = z$ is an arbitrary function $F(u, v) = 0$, where $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are solutions of

[A] $dx = dy = dz$ [B] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ [C] $\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$ [D] none

[6] The necessary and sufficient condition that a pfaffian differential equation $\overline{X}.d\overline{r} = 0$ is integrable is that

[A] $\overline{X}.curl(\overline{X}) = 0$ [B] $\overline{X}.grad(\overline{X}) = 0$

[C] $\overline{X}.curl(\overline{X}) \neq 0$ [D] $\overline{X}.grad(\overline{X}) \neq 0$

[7] Integral surface of the linear partial differential equation $x^2p - y^2q = z^2$ can be obtained by solving the differential equation

[A] $\frac{dx}{z^2} = -\frac{dy}{x^2} = \frac{dz}{y^2}$ [B] $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

[C] $\frac{dx}{y^2} = -\frac{dy}{z^2} = \frac{dz}{x^2}$ [D] $\frac{dx}{x^2} = -\frac{dy}{y^2} = \frac{dz}{z^2}$

[8] A surface orthogonal to given system of surfaces cuts them at an angle measuring

[A] π [B] $\frac{\pi}{2}$ [C] $\frac{\pi}{3}$ [D] $\frac{\pi}{6}$

[9] The differential equation $z = px + qy + p^4 + q^4$ is called a

[A] Clairaut's equation
[B] Linear Partial differential equation
[C] Pfaffian differential equation
[D] Homogeneous differential equation

[10] The complete integral of $z = px + qy + p^2 - q^2$ is given by ---- , where a and b are arbitrary constants.

[A] $z = ax + by$ [B] $z = ay + bx + a^2 - b^2$
[C] $z = a^2x - b^2y$ [D] $z = ax + by + a^2 - b^2$

Q: 2. Answer any TEN of the following. **20**

[1] Find the integral curves of the equations $\frac{dx}{x^2} = -\frac{dy}{y^2} = \frac{dz}{z^2}$

[2] Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$

[3] Find the integral curves of the equations $x.dx = y.dy = z.dz$

[4] Solve : $xp + yq = 4z$

[5] Eliminate the arbitrary function f from the function $z = f(x - y)$

[6] Examine whether $ax - by + z = 7$ is a solution of $px + qy - z + 7 = 0$ or not.

[7] Obtain a differential equation of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dy}{R}$ whose solution generates surfaces orthogonal to the surfaces $x^2 - y^2 - z^2 = c$

[8] Verify that the equation $z = \sqrt{2x+a} + \sqrt{2y+b}$ is the complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$

[9] Obtain integral curve of the linear partial differential equation $px + qy^2 = z^3$

[10] Find the general solution of $(2D + 3D')z = 0$.

[11] Find the complete integral of $(p + q)(z - px - qy) = 1$

[12] Find the Charpit's equations for $p^3 + q^3 = 4$

Q: 3 [A] Solve : $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2 - x^2 - y^2}$ **5**

[B] Solve : $\dfrac{dx}{y^2(x-y)} = \dfrac{dy}{-x^2(x-y)} = \dfrac{dz}{z(x^2+y^2)}$ 5

OR

Q: 3 [A] Solve : $\dfrac{dx}{x+z} = \dfrac{dy}{y} = \dfrac{dz}{z+y^2}$ 5

[B] Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of the fix curves intersecting with planes parallel to the plane XOY 5

Q: 4 [A] Prove that a necessary and sufficient condition that there exists, between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v)=0$ not involving x and y explicitly is that $\dfrac{\partial(u,v)}{\partial(x,y)} = 0$ 5

[B] Determine whether the Pfaffian differential equation $yzdx+2xzdy-3xydz=0$ is integrable or not. Find its solution if it is integrable 5

OR

Q: 4 [A] If $f(u,v)=0$ is a relation between u and v, where u and v are functions of x,y,z and z is a function of x and y then prove that partial differential equation of the relation is given by

$$\frac{\partial(u,v)}{\partial(y,z)}p + \frac{\partial(u,v)}{\partial(z,x)}q = \frac{\partial(u,v)}{\partial(x,y)}$$ 5

[B] Solve : $x^2\dfrac{\partial z}{\partial x} + y^2\dfrac{\partial z}{\partial y} = (x+y)z$ 5

Q: 5 [A] Find the integral surface of the equation $x^2p+y^2q=-z^2$ which passes through the hyperbola $xy=x+y, \ z=1$ 5

[B] Find the surface which is orthogonal to the surface $z(x+y)=c(3z+1)$ and which passes through the circle $x^2+y^2=1, \ z=1$ 5

OR

Q: 5 [A] Find the general integral of the linear partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also the particular integral which passes through the lines $x=1$, and $y=0$ 5

[B] Find the surface which is orthogonal to one parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbolas $x^2-y^2=a^2; \ z=0$ 5

Q: 6 [A] Prove that the equations $f(x,y,p,q)=0$ and $g(x,y,p,q)=0$ are compatible if $\dfrac{\partial(f,g)}{\partial(x,p)} + \dfrac{\partial(f,g)}{\partial(y,q)} = 0$ and verify that the equations $p=P(x,y)$ and $q=Q(x,y)$ are compatible if $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ 5

[B] Find the complete integral of $pq=1$ 5

OR

Q: 6 [A] Find the complete integral of $z=p^2-q^2$ 5

[B] Verify that the partial differential equation $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = \dfrac{2z}{x^2}$ is satisfied by the function $z = \dfrac{1}{x}\phi(y-x) + \phi'(y-x)$ 5

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