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[24/A-16] **SEAT No.________**

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - IV) EXAMINATION - 2019
Thursday , 11^{th} April , 2019
MATHEMATICS : US04CMTH01
(LINEAR ALGEBRA)

Time : 10:00 a.m. to 1:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) .... is a subspace of vector space V .

(a) { 1 } (b) 1 (c) 0 (d) {0}

(2) If S is nonempty subset of vector space V then [S] is .... subspace of V containing S.

(a) largest (b) smallest (c) not (d) unique

(3) $[\phi] =$ ....

(a) 0 (b) $\{0\}$ (c) ϕ (d) V

(4) $\{x^2 - 1 \ , \ x + 1 \ ,$ ....$\}$ is LD set .

(a) $3x - 3$ (b) $2x - 1$ (c) $x - 1$ (d) $x^2 - x - 2$

(5) Any set containing zero vector is .... set .

(a) LI (b) LD (c) empty (d) neither LI nor LD

(6) dim P_n = ...

(a) n (b) 1 (c) n + 1 (d) n - 1

(7) Dimension of $\mathbb{C}$ over $\mathbb{R}$ is ....

(a) 1 (b) 2 (c) 3 (d) 0

(8) $T : V_3 \to V_1$ defined by $T(x_1, x_2, x_3) =$ .... is linear map .

(a) $x_1 x_2$ (b) $x_1^2 + x_2$ (c) $x_1 x_2 x_3$ (d) $x_1 + x_2$

(9) If $T : V_1 \to V_3$ defined by $T(x) = (x, 2x, 3x)$ then ....

(a) $T(x + y) \neq T(x) + T(y)$ (b) $T(\alpha x) \neq \alpha T(x)$ (c) T is not linear (d) T is linear

(10) If a linear map $T : V_2^{\mathbb{C}} \to V_2^{\mathbb{C}}$ defined by $T(z_1, z_2) = (z_1 + iz_2, z_1 - iz_2)$, $B_1 \& B_2$ are standard basis for $V_2^{\mathbb{C}}$ then $(T : B_1, B_2) =$ ....

(a) $\begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}$ (b) $\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}$; (c) $\begin{bmatrix} 1 & 1 \\ i & i \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$

(P.T.O)

Que.2 Attempt the following (Any Ten) 20

(1) For any vector space V, prove that $\alpha \bar{0} = \bar{0}$,$\forall$ scalar α .

(2) In any vector space V , prove that $\alpha u = \alpha v \Rightarrow u = v$, $\forall$ scalar $\alpha \neq 0$, $u, v \in V$.

(3) Is $\{p \in P$ / degree of p $= 4 \}$ subspace of P ? Verify it .

(4) Is $\{(1,0,0),(1,1,1),(1,2,3)\}$ LI ? Verify it .

(5) Is $\{1 + x + 2x^2, 3 - x + x^2, 2 + x, 7 + 5x + x^2\}$ LI ? Verify it .

(6) Determine a value of k that makes the vectors $\{(1,2,k),(0,1,k-1),(3,4,3)\}$ are LD.

(7) Show that the set $\{(1,2),(3,4)\}$ is a basis of V_2 .

(8) Find dimension for subspace $W = \{(x_1, x_2, x_3) \in V_3 / x_1 + x_2 + x_3 = 0\}$ of V_3 .

(9) Is $T : V_3 \to V_3$ defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$ Linear ? Verify it .

(10) Let $T : U \to V$ be a linear map .Then prove that
$T(\alpha_1 u_1 + \alpha_2 u_2 + ... + \alpha_n u_n) = \alpha_1 T(u_1) + \alpha_2 T(u_2) + ... + \alpha_n T(u_n)$,
$\forall$ scalar $\alpha_1, \alpha_2, ..., \alpha_n$, $\forall$ $u_1, u_2, ..., u_n \in U$.

(11) If T_1 and T_2 are linear map from U to V then prove that $T_1 + T_2$ is also linear map .

(12) Determine a linear map T Such that $T : V_2 \to V_2$ defined by $T(1,2) = (2,4)$, $T(2,1) = (2,1)$.

Que.3 (a) Let R^+ be the set of all positive real numbers . Define the operations as bellow : 5
$u + v = uv$, $\forall$ $u, v \in R^+$; $\alpha u = u^\alpha, \forall$ $u \in R^+, \alpha \in \mathbb{R}$.
Prove that R^+ is a real vector space .

(b) If $S = \{(1,-3,2),(2,-1,1)\}$ is a subset of V_3 , then show that $(1,7,-4) \in [S]$ but $(2,-5,4) \notin [S]$. 5

OR

Que.3 (c) Show that $V_3 = \{(x_1, x_2, x_3)$ / $x_1, x_2, x_3 \in \mathbb{R}\}$ is a real vector space under usual addition and scalar multiplication. 5

(d) A nonempty subset S of a vector space V is a subspace of V iff the following conditions are satisfied (i) $u + v \in S$ $\forall$ $u, v \in S$ (ii) $\alpha u \in S$, $\forall$ $u \in S$, $\forall$ scalar α. 5

Que.4 (a) Determine whether the set $S = \{(1,1,0),(0,1,1),(1,0,-1),(1,1,1)\}$ of V_3 is LD or not. If S is LD then locate one of the vectors that belongs to the span of previous ones . Also find a LI subset A of S such that [A]=[S]. 5

(b) Let V be a vector space then prove that 5
(i) $\{v\}$ is LD iff $v = 0$
(ii) $\{v_1, v_2\}$ is LD iff v_1 and v_2 are collinear .
(iii) $\{v_1, v_2, v_3\}$ is LD iff $v_1, v_2 \& v_3$ are co-planer .

OR

Que.4 (c) Determine whether the subset $S = \{(1,1,2),(-3,1,0),(4,0,2),(1,-1,1)\}$ of V_3 is LD or not. If set S is LD then locate one of the vectors that belongs to the span of previous ones .Also find a LI subset A of S such that [A]=[S] . 5

(d) If u , v , w are LI vectors in a vector space V then prove that the sets $u + v, v + w, w + u$ and $u + v, u - v, u - 2v + w$ are also LI . 5

Que.5 (a) Is $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ froms a basis for vector space V_3 ? If not , Determine the dimension of subspace [S] of V_3 . 4

(b) In a vector space V , If $B = \{v_1, v_2, ..., v_n\}$ span V then prove that the following two conditions are equivalent 6
(i) B is LI .
(ii) If $v \in V$, then the expression $v = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n$ is unique .

OR

Que.5 (c) Is $S = \{x-1, x^2+x-1, x^2-x+1\}$ froms a basis for vector space P_2 ? If not , Determine the dimension of subspace [S] of P_2 . 4

(d) If U and W are subspaces of a finite dimensional vector space V then prove that $dim(U+W) = dim(U) + dim(W) - dim(U \cap W)$. 6

Que.6

(a) Let a linear map $T : V_2 \to V_2$ be defined by $T(x,y) = (x,-y)$. Find $(T : B_1, B_2)$,where 7
(i) $B_1 = \{e_1, e_2\}$; $B_2 = \{(1,1),(1,-1)\}$.
(ii) $B_1 = \{(1,1),(1,0)\}$; $B_2 = \{(2,3),(4,5)\}$

(b) Let $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \\ 1 & 2 & -2 \end{bmatrix}$. Determine a linear map $T : V_3 \to V_4$ such that $A = (T : B_1, B_2)$,where $B_1 \& B_2$ are standard bases for V_3 and V_4 respectively . 3

OR

Que.6 (c) Let a linear map $T : V_3 \to V_2$ be defined by $T(e_1) = 2f_1 - f_2$, $T(e_2) = f_1 + 2f_2$, $T(e_3) = 0$, where $\{e_1, e_2, e_3\}$ and $\{f_1, f_2\}$ are standard bases for V_3 and V_2 respectively .Then find matrix of T relative to the bases $B_1 = \{(1,1,0),(1,0,1),(0,1,1)\}$ & $B_2 = \{(1,1),(1,-1)\}$. 5

(e) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$. Determine a linear map $T : V_2 \to V_3$ such that $A = (T : B_1, B_2)$,where $B_1 = \{(1,1),(-1,1)\}$; $B_2 = \{(1,1,1),(1,-1,1),(0,0,1)\}$. 5

—X—