No. of printed pages : 3

[2/A-6] Seat No. ________

SARDAR PATEL UNIVERSITY
B.Sc. (IV Semester) Examination
Wednesday, 25th April, 2018
10.00 am - 1.00 pm
USC04CSTA02 : Statistics
(Probability Distributions)

Total Marks: 70

Note: (*i*) Simple/Scientific calculator is allowed (*ii*) Statistical table will be allowed/ provided on request.
(*iii*) Q.3 to 6 each sub questions is of 5 marks.

Q.1 Multiple Choice Questions (10×1)

(1) The probability distribution in which the probability at each successive draw varies
(a) Geometric (b) Binomial (c) Negative Binomial (d) Hypergeometric

(2) Which of the following distribution has all odd order moments zero?
(a) Chi-square (b) Normal (c) Student's - t (d) Both (b) and (c)

(3) Let X and Y be two independent standard normal variates, the value of $P(X-Y<1)$ equals
(a) 0 (b) 0.5 (c) 1 (d) $P(X+Y<1)$

(4) Let X have an exponential distribution with mean 10, then $P(X>30/X>10)$ is equal to
(a) $1-e^{-2}$ (b) e^{-2} (c) $1-e^{-3}$ (d) e^{-3}

(5) Let X be a continuous r.v. with pdf
$f(x)=\frac{1}{2}, -1 \le x \le 1$ *and zero, otherwise*. The value of $P\left(X^2 \ge \frac{1}{4}\right)$ is
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

(6) Let $X \sim N(0,1)$, what is the value of c so that $P(|X| \le c)=0.99$?
(a) 2.58 (b) 2.33 (c) 1.645 (d) 1.96

(7) If $X_i, i=1,2,\ldots n$ be n independent standard normal variates then $\sum_{i=1}^{n} X_i^2$ has
(a) $N(0,1)$ (b) $N(0,n)$ (c) χ_1^2 (d) χ_n^2

(8) Let X be a continuous random variable with pdf $f(x)=\frac{1}{2}e^{-\frac{x}{2}}, x>0$ *and zero otherwise* then
$P_{25}=$ ______
(a) 1.3863 (b) 2 (c) 0.5754 (d) − 0.8109

(9) If $X \sim N(\mu, \sigma^2)$ then $\left(\frac{X-\mu}{\sigma}\right)^2$ has
(a) $N(0,1)$ (b) χ_1^2 (c) χ_n^2 (d) $N(0,\sigma^2)$

(10) If $f(x)=\frac{K}{20}, 1<x<21$ *and zero otherwise*, is the pdf of X then $V(X)=$ ______
(a) 1.11 (b) 11 (c) 21.21 (d) 33.33

Q.2 Short Type Questions (Attempt Any Ten) (10×2)

(1) Let X be a normal variate with mean 200 and $P(X>225)=0.1587$, Determine $P(X<175)$.

(2) Let X be the no. of customers that enter a bank in an hour is a Poisson variate with $P(X=0)=0.05$. Determine mean and variance of X.

(3) If $X \sim N(7,4)$, what is $P(15.364 \le (X-7)^2 \le 20.095)$?

(4) If $f(x)=\frac{1}{25}, x=1,2,\ldots 25$ *and zero otherwise*
Show that $4(Mean)=Variance$.

(5) Show that the sum of two independent binomial variates is a binomial variate.

(6) The m.g.f of a r.v X is $M(t)=\left\{\frac{1-0.4e^t}{0.6}\right\}^{-5}$

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(i) *State* $E(X), V(X) (ii)$ Determine $P(|X| \leq 2)$

(7) Let X be a chi - square variate with variance 14. Determine $'a'$ and $'b'$ so that $P(a < X < b) = 0.99$?

(8) The probability that a person can hit the target in any trial is 2/3. Find the probability that he will hit the target first time on or before 100^{th} hit.

(9) Let X be a continuous random variable with pdf

$f(x) = 6x(1-x), 0 < x < 1$ *and zero otherwise*. Calculate $P\left(\left|X - \frac{1}{2}\right| > \frac{1}{4}\right)$

(10) Two independent r.v's X_1 and X_2 have Poisson distribution with means m_1 and m_2 respectively. What is the mean and variance of $X_1 + X_2$? State clearly, the result you have used.

(11) The travel time for a businessman travelling from Anand to Baroda is uniformly distributed between 40 and 90 minutes. Find the probability that he will finish her trip in 80 minutes or less.

(12) Let $X \sim F_{(6,9)}$, what is the probability that X is less than or equal to 0.2439?

Q.3(a) A box contains N items of which D items are defectives. A sample of n items is selected at random. What will be the distribution of X where X is the no. of defective items in the selected sample? Also find mean and variance of X.

(b) In an interview process seeking many workers for a new plant, job candidates are interviewed. Suppose each has the same chance of being hired, namely $p = 0.3$.

(i) Let X be the number of candidates interviewed before the first offer is made. Find the probability function of X (ii) What is the probability that the first offer will be made to the eighth candidate?

(iii) What is the probability that the fourth offer will be made to the ninth candidate?

OR

Q.3(a) Obtain mean and variance of Binomial distribution.

(b) In a lottery of 50 tickets numbered 1 to 50, two tickets are drawn simultaneously. Find the prob. that (i) Both tickets have prime numbers. (ii) One prime and one non - prime numbers. (iii) None of the tickets drawn have prime numbers.

Q.4(a) Let $X \sim U(a, b)$. Find the mean, variance and m.g.f. of X.

(b) 2% of light-bulbs have lifetime greater than 1235 hours and 2% have lifetimes less than 765 hours. Assuming lifetime is normally distributed, what are the mean and standard deviation of this distribution? What is the prob. that a randomly selected light-bulb will have lifetime less than 1250 hours?

OR

Q.4(a) Define Normal distribution. Obtain its m.g.f and hence show that $\beta_1 = 0$ and $\beta_2 = 3$.

(b) The length X of a component produced by a machine has probability density function given by

$f(x) = k(1-x), 0 < x < 1$ *and zero otherwise, where k is constant*

(i) Find the value of k (ii) Determine the mean of X (iii) Show that the prob. that the length of a randomly chosen component will be less than the mean is $5/9$.

Q.5(a) Let $Xi, i = 1, 2, \ldots n$ be n independent $N(\mu, \sigma^2)$ variates. Find the distribution of $\sum_{i=1}^{n} aiXi$ where $ai's$ are non – zero constants hence show that $\bar{X}$ has $N\left(\mu, \frac{\sigma^2}{n}\right)$.

(b) A die is rolled independently 120 times. Approximate the probability that (i) More than 42 rolls are odd numbers (ii) the number of two's and three's is from 40 to 45 times.

OR

Q.5(a) Let $Z_1, Z_2, \ldots, Z_7$ be a r.s .of size $n = 7$ from the standard normal distribution $N(0, 1)$. Let $V = Z_1^2 + Z_2^2 + \cdots + Z_7^2$. What will be the distribution of V? Find a and b so that $P(a < V < b) = 0.95$

(b) The m.g.f. of a r.v. X is $M_X(t) = e^{32(e^t - 1)}$

(i) Name the distribution of X (ii) Approximate the following probabilities:

(a) $P(X \leq 22)$ (b) $P(27 \leq X \leq 45)$ (c) $P(|X| > 32)$

Q.6(a) If $X_1, X_2, \dots X_n$ denote a r.s. of size n from a population having mean μ and variance σ^2, then show that $V(\bar{X}) = \frac{\sigma^2}{n}$

(b) Let X has t- distribution with 8 d.f. Determine the value of K such that
$(i)\ P(|X| > K) = 0.05\ (ii)\ P(X^2 < K) = 0.99\ (iii)\ P(-K < X < K) = 0.9.$

OR

Q.6(a) Let $X_1, X_2, \dots, X_n$ be a r.s. of size $n = 16$ from the normal distribution $N(50, 100)$, determine
$(i)\ P(796.2 \leq \sum_{i=1}^{16} (Xi - 50)^2 \leq 2630)\ (ii)\ P(726.1 \leq \sum_{i=1}^{16} (Xi - \bar{X})^2 \leq 2500)$

(b) Let $\bar{X}$ and S^2 be the sample mean and variance associated with a r.s. of size $n = 16$ from a normal distribution with mean μ and variance 225.
(i) Find the constants a and b so that $P(a \leq S^2 \leq b) = 0.95$
(ii) Find a constant k so that $P\left(-k \leq \frac{\bar{X}-\mu}{S} \leq k\right) = 0.95, where\ \bar{X} = \frac{1}{n}\sum Xi\ and\ S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$

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