[10/A-12] SEAT No.________ No. of Printed Pages : 3

Sardar Patel University, Vallabh Vidyanagar

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Subject : Mathematics US04CMTH02 Max. Marks : 70

Differential Equations

Date: 23/04/2018 Timing: 10:00 am - 01:00 pm

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] Integral curve of $e^x dx = e^y dy = e^z dz$ is given by

[A] $e^x + e^y = c_1$; $e^y - e^z = c_2$ [B] $e^x - e^y = c_1$; $e^y + e^z = c_2$
[C] $e^x + e^y = c_1$; $e^y + e^z = c_2$ [D] $e^x - e^y = c_1$; $e^y - e^z = c_2$

[2] The equations $x - y = c_1; y + z = c_2$, represent integral curve of

[A] $dx = dy = -dz$ [B] $dx = dy = dz$ [C] $dx = -dy = dz$ [D] $-dx = dy = dz$

[3] Integral curve of $2xdx = dy = 2zdz$ is given by

[A] $x^2 + y = c_1, y + z^2 = c_2$ [B] $x^2 + y = c_1, y - z^2 = c_2$
[C] $x^2 + y = c_1, y + z^2 = c_2$ [D] $x^2 - y = c_1, y - z^2 = c_2$

[4] The diferential equation dx+2dy+3dz=0 is a ____ differential equation.

[A] Pfaffian [B] linear partial [C] non-linear partial [D] none

[5] $ax + by - z = 1$ is a solution of

[A] px-qy-z=1 [B] qx-py-z=1 [C] p+y-z=1 [D] px+qy-z=1

[6] The general solution of the partial differential equation $\frac{p}{x} + \frac{q}{y} = \frac{1}{z}$ is an arbitrary function $F(u, v) = 0$, where $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are solutions of

[A] $xdx + ydy = zdz$ [B] $xdx = ydy = zdz$
[C] $xdx + ydy + zdz = 0$ [D] $\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$

[7] The surfaces orthogonal to a one parameter family of surfaces $x^2 + y^2 + z^2 = c$ are the surfaces generated by the integral curves of the equations

[A] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ [B] $\frac{dx}{z} = \frac{dy}{x} = \frac{dz}{y}$
[C] $\frac{dx}{y} = \frac{dy}{z} = \frac{dz}{x}$ [D] $xdx = ydy = zdz$

[8] A surface orthogonal to given system of surfaces cuts them at an angle measuring

[A] π [B] $\frac{\pi}{2}$ [C] $\frac{\pi}{3}$ [D] $\frac{\pi}{6}$

[9] If two partial differential equations have a common solution then they are

[A] Pfaffian equations [B] Compatible [C] Non-compatible [D] none

[P.T.O.]

[10] For a partial differential equation $pq = 5$, the Charpit's auxiliary equations are given by

[A] $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$ [B] $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$

[C] $\frac{dx}{q} = \frac{dy}{p} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$ [D] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$

Q: 2. Answer any TEN of the following. **20**

[1] Find the integral curves of $\frac{dx}{2} = -\frac{dy}{3} = \frac{dz}{4}$

[2] Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$

[3] Find the integral curves of $\frac{dx}{x^3} = -\frac{dy}{y^3} = \frac{dz}{z^3}$

[4] Eliminate the arbitrary function f from the function $z = x + y + f(xy)$

[5] Determine whether the equation $ydx + xdy = 5zdz$ is integrable or not.

[6] Examine whether $ax^2 + by^2 - z = 1$ is a solution of $px + qy - 2z = 2$ or not.

[7] Obtain a differential equation of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dy}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $x^2 - y^2 - z^2 = c$

[8] Obtain integral curve of the linear partial differential equation $px + qy^2 = z^3$

[9] Verify that the equation $z = \sqrt{2x+a} + \sqrt{2y+b}$ is the complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$

[10] Find the general solution of $(2D + 3D')z = 0$.

[11] Find the complete integral of $(p+q)(z - px - qy) = 1$

[12] Find the Charpit's auxiliary equations for $5p^2q^2 = 1$

Q: 3 [A] Solve : $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$ **5**

[B] Solve : $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2 - x^2 - y^2}$ **5**

OR

Q: 3 [A] Solve : $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}$ **5**

[B] Find the equation of system of curves on the cylinder $2y = x^2$ orthogonal to each intersection with the hyperboloid of one parameter system $xy = z + c$ **5**

Q: 4 [A] Prove that a necessary and sufficient condition that there exists, between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$ not involving x and y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$ 5

[B] Determine whether the Pfaffian differential equation

$$(y+z)dx + (z+x)dy + (x+y)dz = 0$$

is integrable or not. Find its solution if it is integrable 5

OR

Q: 4 [A] Prove that the general solution of the linear differential equation $pP+qQ = R$ is $F(u, v) = 0$, where $F(u, v) = 0$ is an arbitrary function of $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ which form a solution of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ 5

[B] Solve : $y^2p - xyq = x(z - 2y)$ 5

Q: 5 [A] Find the general integral of the linear partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also the particular integral which passes through the lines $x = 1$, and $y = 0$ 5

[B] Find the surface which is orthogonal to one parameter system $z = cxy(x^2+y^2)$ and which passes through the hyperbolas $x^2 - y^2 = a^2;\ z = 0$ 5

OR

Q: 5 [A] Find the integral surface of the equation $x^2p+y^2q = -z^2$ which passes through the hyperbola $xy = x + y,\quad z = 1$ 5

[B] Find the integral surface of the linear partial differential equation $2y(z-3)p+(2x-z)q = y(2x-3)$ passing through the circle $z = 0, x^2+y^2 = 2x$ 5

Q: 6 [A] Find the complete integral of $p + q = pq$ 5

[B] Find the general solution of $(D^2 - DD')z = \cos x \cos 2y$ 5

OR

Q: 6. Define compatible differential equations. Also prove that two first order partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible if $[f, g] = 0$ where

$$[f,g] = \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)}$$

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