SC

SEAT No._________

[A-12]

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER-IV) (JUNE 2010 BATCH) EXAMINATION -2018
Monday , 9^{th} April , 2018
MATHEMATICS : US04CMTH01
(LINEAR ALGEBRA)

Time : 10:00 a.m. to 1:00 p.m. **Maximum Marks:70**

Que.1 Fill in the blanks. 10

(1) In any vector space V , $\alpha\,\bar{0}$ = ...

(a) 0 (b) α (c) 1 (d) $\bar{0}$

(2) .... is a subspace of vector space V .

(a) 1 (b) V (c) 0 (d) $\{V\}$

(3) If S is nonempty subset of vector space V_3 then (0,0,0) .... [S].

(a) = (b) $\neq$ (c) $\in$ (d) $\notin$

(4) Any set containing zero vector is .... set .

(a) LI (b) LD (c) empty (d) neither LI nor LD

(5) Every subset of .... set is LI .

(a) LD (b) LI (c) zero (d) power

(6) {................ , $\cos 2x$, 1 } is LD set .

(a) $\sin^2 x$ (b) $\sin x$ (c) $\cos x$ (d) $\sin 2x$

(7) If B is a basis for V ,then ....

(a) B is LI (b) B is LD (c) $[B] \neq V$ (d) $B = \{0\}$

(8) dim V_3 = ...

(a) 1 (b) 2 (c) 3 (d) 0

(9) If $T : V_1 \to V_2$ defined by $T(x) = (x\ ,\ 0)$ then $T(x+y)$ = ...

(a) $(x\ ,\ y)$ (b) $(x+y\ ,\ 0)$ (c) $(x\ ,\ 0)$ (d) $(y\ ,\ 0)$

(10) If $T : V_1 \to V_3$ defined by $T(x) = (x\ ,\ 2x\ ,\ 3x)$ then ....

(a) $T(x+y) \neq T(x) + T(y)$ (b) $T(\alpha x) \neq \alpha T(x)$ (c) T is not linear (d) T is linear

Que.2 Answer the following (Any Six) . 12

(1) For any vector space V, prove that $\alpha\,\bar{0} = \bar{0}$, $\forall$ scalar α .

(2) For any vector space V, prove that $\alpha\,(u-v) = \alpha\,u - \alpha\,v$, $\forall\ u\ ,\ v \in V$, $\forall$ scalar α .

(3) In V_2 , show that $(3,7) \in [(1,2),(0,1)]$.

(4) If $S = \{(1,-3,2),(2,-1,1)\}$ is a subset of V_3 , then show that $(2,-5,4) \notin [S]$.

(P. T. O.)

(5) Is $\{(1,0,0),(2,0,0),(0,0,1)\}$ LI in V_3 ? Verify it .

(6) Show that the set $\{(1,1,1),(1,-1,1),(0,1,1)\}$ is a basis of V_3 .

(7) Check whether the mapping $T: V_3 \to V_3$ defined by $T(x_1,x_2,x_3)=(x_1,x_2,0)$ is linear or not .

(8) Check whether the mapping $T: V_3 \to V_1$ defined by $T(x_1,x_2,x_3)=x_2$ is linear or not .

Que.3 (a) Let R^+ be the set of all positive real numbers . Define the operations as bellow : $u+v=uv, \forall\ u,v \in R^+$, $\alpha u = u^\alpha, \forall\ u \in R^+, \alpha \in \mathbb{R}$. Prove that R^+ is a real vector space . 8

OR

Que.3 (b) Show that $V_3=\{(x_1,x_2,x_3)\ /\ x_1,x_2,x_3 \in \mathbb{R}\}$ is a real vector space under usual addition and scalar multiplication. 8

Que.4 (a) If u , v , w are LI vectors in a vector space V then prove that 5
(i) $u+v, v+w, w+u$ are also LI .
(ii) $u+v, u-v, u-2v+w$ are also LI .

(b) Check whether the subset $\{(1,2,1),(-1,1,0),(5,-1,2)\}$ of V_3 is LI or LD ? 3

OR

Que.4 (c) Let $S=\{(1,2,1),(1,1,-1),(4,5,-2)\}$ .Determine which of the following vectors are in [S] . 5
(i) $(1,1,0)$ (ii) $(2,-1,-8)$

(d) Check whether the subset $\{(1,1,0),(1,-1,0),(1,1,-1)\}$ of V_3 is LI or LD ? 3

Que.5 (a) Determine whether the set $S=\{(1,1,2),(-3,1,0),(4,0,2),(1,-1,1)\}$ is LD ? If set S is LD then locate one of the vectors that belongs to the span of previous ones . Also find a LI subset A of S such that [A]=[S]. 5

(b) Let V be a vector space and $dim(V)=n$ then prove that any set of $n+1$ vectors of V is LD . 3

OR

Que.5 (c) Determine whether the set $S=\{(1,-1,2),(1,1,2),(3,0,0),(2,1,-1)\}$ is LD ? If set S is LD then locate one of the vectors that belongs to the span of previous ones . Also find a LI subset A of S such that [A]=[S]. 5

(d) In any vector space V , prove that Every superset of LD set is also LD . 3

Que.6 (a) Let the set $\{v_1,v_2,\ldots,v_k\}$ be a linearly independent subset of an n - dimensional vector space V . then prove that we can find vectors $\{v_{k+1},v_{k+2},\ldots,v_n\}$ such that the set $\{v_1,v_2,\ldots,v_k,v_{k+1},\ldots,v_n\}$ is a basis for V . 4

(b) Is the subset $\{(1,1,1),(1,0,-1),(3,-1,0),(2,1,-2)\}$ from a basis for V_3 ? If not , then find a basis for [S] . 4

OR

Que.6 (c) If U and W are subspaces of a finite dimensional vector space V then prove that $dim(U+W)=dim(U)+dim(W)-dim(U\cap W)$. 5

(d) Is the subset $\{(1,2,3),(3,1,0),(-2,1,3)\}$ from a basis for V_3 ? If not , then find a basis for [S] . 3

Que.7 (a) Let U and V be a vector space and $T: U \to V$ be any map then prove that T is linear iff $T(\alpha u_1+u_2)=\alpha T(u_1)+T(u_2)$, $\forall$ scalar α , $\forall\ u_1,u_2 \in U$. 4

(b) Check whether a mapping $T: V_2 \to V_2$ defined by $T(x\ ,\ y)=(3x+2y\ ,\ 3x-2y)$ is linear or not . 4

OR

Que.7 (c) Check whether a mapping $T : V_3 \to V_1$ defined by $T(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ is linear or not . 4

(d) Determine a linear map in the cases $T : V_2 \to V_4$ defined by $T(1,1) = (0,1,0,0)$, $T(1,-1) = (1,0,0,0)$. 4

Que.8 (a) Let a linear map $T : V_2 \to V_2$ be defined by $T(x,y) = (x,-y)$. Find $(T : B_1, B_2)$,where $B_1 = \{e_1, e_2\}$; $B_2 = \{(1,1),(1,-1)\}$. 4

(b) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine a linear map $T : V_3 \to V_3$ such that $A = (T : B_1, B_2)$, where $B_1 = \{(1,1,1) , (1,0,0) , (0,1,0)\}$; $B_2 = \{(1,2,3) , (1,-1,1) , (2,1,1)\}$. 4

OR

Que.8 (c) Let a linear map $T : V_2 \to V_2$ be defined by $T(x,y) = (x,-y)$. Find $(T : B_1, B_2)$,where $B_1 = \{(1,1) , (1,0)\}$; $B_2 = \{(2,3) , (4,5)\}$. 4

(d) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine a linear map $T : V_3 \to V_3$ such that $A = (T : B_1, B_2)$,where $B_1 = B_2 = \{e_1, e_2, e_3\}$. 4

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