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(17)

SARDAR PATEL UNIVERSITY  
B.Sc. (SEM-IV) Examination(Regular & NC)  
Wednesday, 13<sup>th</sup> April, 2016  
USO4EMTHO5: (Calculus and Algebra-2)

Time: 10:30 a.m. to 12:30 p.m.

Maximum Marks : 70

Note: Figures to the right indicate marks to the questions.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

- (1) If  $AC-B^2 = 0$ , Then \_\_\_\_\_  
 (a) local minimum and local maximum of f do not exist (b) we can not conclude anything  
 (c) f(a,b) is a local maximum of f (d) f(a,b) is a local minimum of f
- (2) If  $f(x,y) \leq f(a,b)$  for all  $(x,y) \in E \subset R^2$ , then f is said to be \_\_\_\_\_  
 (a) local extreme at (a,b) (b) global minimum at (a,b)  
 (c) global maximum at (a,b) (d) constant value at (a,b)
- (3) If  $f(x,y) = x^3 + y^3$ ,  $\text{grad} f =$  \_\_\_\_\_  
 (a)  $3x^2\bar{i} + 2y\bar{j}$  (b)  $3x^2\bar{i} - 2y\bar{j}$  (c)  $3x\bar{i} - 2y\bar{j}$  (d)  $3x^2\bar{i} + y^2\bar{j}$
- (4)  $\nabla(\nabla f) =$  \_\_\_\_\_  
 (a) 0 (b)  $(\nabla f)^2$  (c)  $2\nabla f$  (d)  $\nabla^2 f$
- (5) Divergence of  $\vec{v}$  is denoted by \_\_\_\_\_  
 (a)  $\nabla \times \vec{v}$  (b)  $\nabla \vec{v}$  (c)  $\nabla \cdot \vec{v}$  (d)  $\nabla^2 \cdot \vec{v}$
- (6) If  $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$ , then  $\frac{\partial r}{\partial y} =$  \_\_\_\_\_  
 (a)  $\frac{y}{r}$  (b) 1 (c)  $\frac{r}{y}$  (d) y
- (7) If  $\nabla \cdot V = 0$  Then V is called \_\_\_\_\_  
 (a) Laplacian (b) Harmonic (c) Irrotational (d) Solenoidal
- (8)  $\nabla \times (\nabla f) =$  \_\_\_\_\_  
 (a) -1 (b)  $\vec{0}$  (c) 1 (d) 0
- (9) For an element 'a' in Boolean Algebra  $a \cdot 0 =$  \_\_\_\_\_  
 (a) a (b) 1 (c) 0 (d) a'
- (10) If  $a \in B$ . Then  $a+a' =$  \_\_\_\_\_  
 (a) a (b) a' (c) 0 (d) 1

Q.2 Answer ANY TEN of the following: [20]

- (1) Define stationary point.
- (2) Find A, B, C for the function  $x^4 - 2x^2 - 2y^2 + 4xy + y^4$ .
- (3) Find stationary point of  $y^2 + x^2y + x^4$
- (4) Define : Gradient of scalar field.
- (5) Find Gradient of a function  $f(x,y) = \log r$ , where  $\vec{r} = x\bar{i} + y\bar{j}$
- (6) Prove that  $\nabla(f+g) = \nabla f + \nabla g$ .
- (7) Define: The divergence of a vector field.
- (8) Prove that  $\nabla \times (f\vec{v}) = f(\nabla \times \vec{v}) + \nabla f \times \vec{v}$ .
- (9) Define : The curl of a vector field.
- (10) Write distribution law for Boolean Algebra.
- (11) Write any two laws of compliment in Boolean Algebra.
- (12) Draw the network represented by following Boolean algebra.  
 $x(xy + x' + xy')$

(1)

(P.T.O.)

Q.3 Investigate the maxima and minima of the function  $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ . (10)

OR

Q.3 A rectangular box open at the top is to have a volume of  $32 \text{ m}^3$ . Find the dimension of box so that the total surface area is minimum. (10)

Q.4

(a) Find gradient of the function  $f(x, y) = (x^2 + y^2 + z^2)^2$  at  $(1, 2, 3)$ . (5)

(b) Show that the function  $f(x, y) = \frac{x}{(x^2 + y^2)}$  is a harmonic function. (5)

OR

Q.4

(a) Find directional derivative of  $f(x, y, z) = 4xz^3 - 3x^2y^2z$  at  $(2, -1, 2)$  in the direction of  $\vec{a} = 2\vec{i} - 3\vec{j} + 6\vec{k}$ . (5)

(b) Find unit normal vector to the surface  $z^2 = (x^2 + y^2)$  at  $(3, 4, 5)$ . (5)

Q.5

(a) Show that  $\nabla(\vec{r}^n \cdot \vec{r}) = (n+3)\vec{r}^n$  where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ ,  $r = |\vec{r}|$ . (5)

(b) Show that  $\nabla(\nabla \times \vec{v}) = 0$ . (5)

OR

Q.5

(a) Verify  $\nabla(f\vec{v}) = f(\nabla\vec{v}) + \vec{v}\nabla f$  for  $f = e^{xyz}$  and  $\vec{v} = ax\vec{i} + by\vec{j} + cz\vec{k}$ . (5)

(b) Show that  $\nabla\left(\frac{r}{r^3}\right) = 0$ . (5)

Q.6

(a) In every Boolean algebra (B) each of the binary operations (+) and ( $\cdot$ ) are associative. (5)

(b) Simplify following function and draw the network represented by them. (5)

(1)  $1+x$  (2)  $x \cdot 0$  (3)  $x + xy'$

OR

Q.6

(a) State and prove De-Morgan's law for Boolean algebra (B). (5)

(b) Find Boolean function for the following circuit. Simplify it and draw simplified circuit. (5)

