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SARDAR PATEL UNIVERSITY

B.Sc. EXAMINATION (SEMESTER: IV)

2016

April, 18th 2016.

Monday

Subject: Probability Distributions

Subject code: US04CSTA02

Time: 10.30 a.m. to 01.30 p.m. **Marks: 70**

1 **Multiple Choice Questions** [10]

(1) For a binomial distribution with n = 7 and p = 0.50, we get ------

(a) P(X = 3) < P(X = 4) (b) P(X = 3) > P(X = 4)
(c) P(X = 3) = P(X = 4) (d) P(X = 3) ≠ P(X = 4)

(2) If $M_X(t) = e^{3(e^t-1)}$ is the m.g.f. of a random variable X then X ~----- distribution

(a)binomial (b)Poisson (c)geometric (d) none

(3) If $f(x) = \frac{1}{21}$, x = 1,2,......21.
= 0 , otherwise; is the p.m.f. of X then P(3 < X < 9) = -------.

(a) $\frac{5}{21}$ (b) $\frac{8}{21}$ (c) $\frac{2}{21}$ (d) none

(4) If f(x) = kx^2 , 0 < x < 1.
= 0 , otherwise ; is the p.d.f. of X then k = ---------.

(a) 1 (b) 2 (c) 3 (d) 4

(5) If $M_X(t) = e^{25t(1+t)}$ is the m.g.f. of a continuous random variable X then X ~ ------ distribution.

(a) exponential (b) gamma (c) normal (d) none

(6) If $M_X(t) = \frac{e^{3t} - e^{2t}}{t}$, t ≠ 0, is the m.g.f. of a random variable X then X ~ ----------- distribution

(a) normal (b) exponential (c) continuous uniform (d) none

(7) If X_1 and X_2 are two independent N(5,2) and N(2,1) distributions respectively then the distribution of Y = X_1 - X_2 follows --------- distribution.

(a) N(3,2) (b) N(3,3) (c) N(3,4) (d) none

(8) If X_1 and X_2 are two independent b(5,0.25) and b(4,0.25) distributions respectively then the distribution of Y = X_1 + X_2 follows ---------- distribution.

(a) b(9 , 0.25) (b) b(9 , 0.20) (c) b (9, 0.10) (d) none

(9) If $S^2 = \frac{1}{n}\sum_1^n(X_i - \bar{X})^2$ is the sample variance of a random sample from N(μ , σ^2) then $\frac{nS^2}{\sigma^2}$ ~ --------- distribution.

(a) $\chi^2_{(n)}$ (b) $\chi^2_{(n-1)}$ (c) $\chi^2_{(1)}$ (d) none

(10) If X ~ $F_{(r_1, r_2)}$ then $\frac{1}{X}$ ~ ------- distribution.

(a) $F_{(r_2, r_1)}$ (b) $F_{(r_1 * r_2)}$ (c) $F_{(r_1/r_2)}$ (d) none

2 **Short Questions : (Attempt any TEN) : Two Marks each.** [20]

(1) Obtain recurrence relation for the Binomial distribution.

(2) If $P(t) = e^{2(t-1)}$ is the p.g.f. of a random variable X then find P(X = 0), P(X > 1) and mean.

(3) If $M_X(t) = (1 - 2t)^{-1}$ is the m.g.f. of a random variable X then identify the distribution of X and state its mean, variance and write its p.d.f.

(4) If $f(x) = \frac{1}{6}, -3 < x < 3$;
$= 0$, otherwise, is the p.d.f. of X then find mean and variance.

(5) If $X \sim N(25, 25)$ distribution then find (i) $P(20 \le X \le 40)$ and $P(|X - 25| < 5)$.

(6) If $f(x) = k\,x(1 - x)$, $0 < x < 1$ and zero otherwise ,is the p.d.f of X then find (i) k and (ii) $P(X < 0.50)$.

(7) If X and Y follow independently P(2) and P(1) distribution. What is the distribution of Z = X + Y. State the result you use.

(8) If $X \sim b(100, 0.50)$ distribution then find approximately $P(X = 50)$ and $P(55 \le X \le 65)$.

(9) If $X \sim P(64)$ then find $P(X = 80)$. State the result you used.

(10) If $X \sim N(0, 1)$ distribution then find $P(X^2 > 3.841)$ and $P(X^2 < 3.841)$.

(11) Define F – distribution. Write the p.d.f. of $F_{(2, 2)}$ distribution.

(12) If $X_1, X_2, \ldots\ldots X_n$ denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ then show that $E(\bar{X}) = \mu$.

3 (a) Define discrete uniform distribution. Find its mean and variance. [5]

(b) The number of arrivals per hour at an automatic teller machine (ATM) follows Poisson distribution with a mean of 2 arrivals/hour. What is the probability that (i) more than three (ii) two or less (iii) Exactly 2, arrivals occur in an hour? [5]

OR

3 (a) Define negative binomial distribution. Obtain its mean and variance. [6]

(b) It was claimed that 1 out of 4 dentists recommend sensodyne tooth paste to his patients to prevent cavities. Suppose that the claim is true. If 10 dentists are selected independently and at random, let X be the number of dentists who recommend sensodyne tooth paste to his/her patients (i) How is X distributed (ii) Give mean and variance of X. (iii) Determine $P(X \ge 4)$, $P(X = 2)$. [4]

4 Define Normal distribution. Obtain its m.g.f. and hence or otherwise find mean and variance. Also find c.g.f. and β_1 and β_2. [10]

OR

4 Define continuous uniform distribution defined over an interval (a, b), a, b ∈ R. Find m.g.f. and hence find mean and variance. Obtain its c.d.f. and find the expression for p^{th} quantile. Hence or otherwise find Q_1, D_5 and P_{75}. [10]

5 (a) If X_1 and X_2 are two independently distributed as N(10 , 25) and N(2 ,144) respectively. Show by using m.g.f. that $Y = X_1 + X_2 \sim N(12,169)$. Hence find $P(Y \le -25)$ and $P(-25 \le Y \le 38)$. [5]

(b) If $X_1, X_2, \ldots X_n$ are independent random variables with m.g.f.s. $Mx_1(t)$, $Mx_2(t)$,....... $Mx_n(t)$ then prove that the m.g.f. of $Y = \sum_1^n X_i$ is given by $My(t) = \prod_1^n Mx_i(t)$. [5]

OR

5 (a) If $X_1, X_2, \ldots\ldots X_n$ are n independent random variables with m.g.f. $M_{X_i}(t) = e^{\lambda_i(e^t - 1)}$, i = 1,2,........,n then find the m.g.f. of $Y = \sum X_i$. Identify the distribution of Y and state its p.d.f. and mean. [5]

(b) If $X_1, X_2, \ldots\ldots X_n$ are n independent $N(\mu, \sigma^2)$ variables then find the m.g.f. of $\bar{X}$ and identify the distribution. [5]

6 (a) If $X_1, X_2, \ldots\ldots X_n$ denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ and $S^2 = \frac{1}{n}\sum_1^n (X_i - \bar{X})^2$ and $S'^2 = \frac{1}{n-1}\sum_1^n (X_i - \bar{X})^2$. Obtain $E(S^2)$ and $E(S'^2)$ [5]

(b) If $X \sim F_{(2,r)}$ distribution, $r \geq 2$ then prove that $P(X \geq k) = (1 + \frac{2k}{r})^{-\frac{r}{2}}$ [5]

OR

6 (a) A random sample of size n = 36 is to be taken from N(15,144) distribution. Find $P(13 \leq \bar{X} \leq 17)$ and $P(\bar{X} \geq 19)$. Also state $E(\bar{X})$, $E(3\bar{X} + 5)$, $V(\bar{X})$ and $V(3\bar{X} + 5)$. [5]

(b) If $X_1, X_2, \ldots\ldots X_n$ denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ then show that $V(\bar{X}) = \frac{\sigma^2}{n}$. [5]

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