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SARDAR PATEL UNIVERSITY

B.Sc. (SEM- II) Examination(Regular & NC)

Tuesday, 12th April-2016

USO4CMTHO2: (DIFFERENTIAL EQUATIONS)

Time: 10:30 a.m. to 01:30 p.m.

Maximum Marks : 70

Note: Figures to the right indicate marks to the questions.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

(1) Orthogonal trajectories of the given curves intersect them at ____ angle.
(a) acute (b) right (c) obtuse (d) none of these

(2) One solution of $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$ is given by________
(a) $xyz = c$ (b) $x^2y^2 = c$ (c) $x^2 + y^2 = c$ (d) $x^2 - y^2 = c$

(3) The solution of $pdx = qdy = rdz$ is given by______
(a) $px + qy = x$ (b) $px - qy = c_1; qy - rz = c_2$
(c) $px - qy = z; qy - rz = x$ (d) $qy - rz = y$

(4) The necessary and sufficient condition that a Pfaffian differential equation $\bar{X}.d\bar{r}$ =0 is integrable is that ______
(a) $\bar{X}.curl\bar{X} \neq 0$ (b) $\bar{X}.grad\bar{X} = 0$ (c) $\bar{X}.curl\bar{X} = 0$ (d) $\bar{X}.grad\bar{X} \neq 0$

(5) Partial differential equation for $z = f(x - y)$ is ____
(a) z=x-y (b) p=q (c) $p^2 - q = 0$ (d) p+q=0

(6) Eliminate the arbitrary function f from z=f(x^2-y^2).
(a) $yp + xq = 0$ (b) $px - qy = x - y$ (c) $xp - yq = 0$ (d) $p + q = 0$

(7) Degree of the partial differential equation $\frac{\partial^3 z}{\partial x^3} + x^2(\frac{\partial^2 z}{\partial y^2})^2 - y\left(\frac{\partial z}{\partial y}\right)^5 = 4$ is
(a) 5 (b) 1 (c) 4 (d) 2

(8) Which of the following is a linear partial differential equation?
(a) $pq = 1$ (b) $p - x^3q = 2z$ (c) $p + q = pq$ (d) $2p^2 = q$

(9) A complete integral of the partial differential equation $z = px + qy + \log pq$ is_____.
(a) $ax + by + logxy = z$ (b) $x + y + \log ab = z$
(c) $z = x^a + y^b + e^{ab}$ (d) $ax + by + \log ab = z$

(10) For linear partial differential equation with constant coefficient $F(D, D')z = f(x, y)$ the operator $D' =$ ____
(a) $\frac{\partial}{\partial x}$ (b) $\frac{\partial}{\partial p}$ (c) $\frac{\partial}{\partial y}$ (d) $\frac{\partial}{\partial q}$

Q.2 Answer ANY TEN of the following: [20]

(1) Solve: $\frac{dx}{1+x} = \frac{dy}{1+y} = \frac{dz}{z}$

(2) Solve: $\frac{dx}{x^2} = \frac{-dy}{y^2} = \frac{dz}{z^2}$

(3) Solve : $xdx = ydy = zdz$.

(4) Determine whether the equation $ydx + xdy - zdz = 0$ is integrable or not.

(5) Eliminate a & b from the $z = ax^3+by^3$.

(6) Obtain partial differential equation for a set of spheres whose centre lies along y axis.

(7) Find the differential equation of the surface which is orthogonal to $x^2 + y^2 + z^2 = cz$.

(8) Find the integral surface of x^2+y = c_1, xz + y = c_2 passes through the line x=0,y=1.

(9) Define : non linear partial differential equation with example.

(10) Find the complete integral of the equation $pq = 1$.

(11) Find C.F. of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

(12) Find the complete integral of the equation $(p + q).(z - xp - yq) = 1$.

Q.3

(a) Solve: $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$ [5]

(b) Solve: $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ [5]

OR

Q.3

(a) Solve: $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$ [5]

(b) Find the equation of the system of the curve on the cylinder $2y = x^2$ orthogonal to each intersection with the hyperboloids of the one parameter system $xy = z + c$. [5]

Q.4

(a) Determine whether the Pfaffian differential equation $2xzdx + zdy - dz = 0$ is integrable or not. Find its solution if it is integrable. [5]

(b) If f(u, v)=0 is a relation between u and v, where u and v are function of x, y, z and z is a function of x and y. Then P.T. partial diff. Eqn. Of the relation is given by $p\frac{\partial(u,v)}{\partial(y,z)} + q\frac{\partial(u,v)}{\partial(z,x)} = \frac{\partial(u,v)}{\partial(x,y)}$. [5]

OR

Q.4

(a) If X is a vector such that $\bar{X}.curl\bar{X} = 0$ and μ is an arbitrary function of x,y and z then prove that $\mu\bar{X}.curl(\mu\bar{X}) = 0$. [5]

(b) Find the general solution of the following linear partial diff. Eqn. $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$ [5]

Q.5

(a) Find the integral surface of linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which passes through the line $x + y = 0, z = 1$. [5]

(b) Show that $(x-a)^2+(y-b)^2+z^2=1$ is the complete integral of the non linear partial differential equation $z^2(1+p^2+q^2)=1$.Determine a general solution by finding the envelope of its particular solution. [5]

OR

Q.5

(a) Find surfaces which is orthogonal to the surface z(x+ y) = c(3z+1) and which passes through the circle $x^2 + y^2 = 1, z = 1$. [5]

(b) Find the integral surface of linear partial differential equation $(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$ which passes through the curve $xz = a^3, y = 0$. [5]

Q.6 Show that the equation $xp = yq$ and the equation $z(xp + yq) = 2xy$ are compatible and solve them. [10]

OR

Q.6 Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfied by the equation $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$. [10]

X= X- X