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SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - IV)(JUNE 2010 BATCH) EXAMINATION - 2016

Mon[redacted]day , 2^{nd} May , 2016

MATHEMATICS : US04CMTH01

(Linear Algebra)

Time : 2:30 p.m. to 5:30 p.m. **Maximum Marks : 70**

Que.1 Fill in the blanks. 10

(1) In any vector space V , $\alpha\, u = \alpha\, v \Rightarrow\ u$ .... v , $\forall u, v \in V, \alpha \in \mathbb{R}, \alpha \neq 0$

(a) $=$ (b) $<$ (c) $>$ (d) $\neq$

(2) .... is a subspace of vector space V .

(a) { 1 } (b) 1 (c) 0 (d) {0}

(3) $[(0,0,2)\ ,\ (0,3,1)] =$ ....

(a) xy-plane (b) yz-plane (c) zx-plane (d) V_3

(4) Any set containing zero vector is .... set .

(a) LI (b) LD (c) empty (d) neither LI nor LD

(5) The vectors $(a, b) \& (c, d)$ of V_2 are LD iff ....

(a) ad = bc (b) ab = cd (c) ac = bd (d) a = c

(6) $\{x^2 - 1\ ,\ x + 1\ ,$ ....$\}$ is LI set .

(a) $1 - x^2$ (b) $x^2 + x$ (c) $x - 1$ (d) $x^2 - x - 2$

(7) dim $P_3 =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

(8) $T : V_3 \to V_1$ defined by $T(x_1, x_2, x_3) =$ .... is not linear map .

(a) $x_1 + x_2 + x_3^2$ (b) $x_1 + x_3$ (c) $x_1 + x_2 + x_3$ (d) $x_1 - x_2$

(9) If $T : V_1 \to V_2$ defined by $T(x) = (x, 0)$ then $T(x + y) =$ ....

(a) (x, y) (b) $(x + y, 0)$ (c) $(x, 0)$ (d) $(y, 0)$

(10) If $T : V_1 \to V_3$ defined by $T(x) = (x, 2x, 3x)$ then ....

(a) $T(x + y) \neq T(x) + T(y)$ (b) $T(\alpha x) \neq \alpha T(x)$ (c) T is not linear (d) T is linear

Que.2 Attempt the following (Any Six) 12

(1) For any vector space V, prove that $\alpha\, \bar{0} = \bar{0}$,$\forall$ scalar α .

(2) For any vector space V, prove that $(-1)u = -u$, $\forall\ u \in V$.

(3) Is the set $\{(1,0,0), (2,0,0), (0,0,1)\}$ LI in V_3 ?

(P.T.O.)

(4) Is the set $\{(1,1,0),(1,-1,0),(1,1,-1)\}$ LD in V_3 ?

(5) Is the set $\{\sin^2 x, \cos 2x, 1\}$ LD ?

(6) Show that the set $\{(1,1,1),(1,-1,1),(0,1,1)\}$ is a basis of V_3 .

(7) Show that the map $T : V_3 \to V_3$ defined by $T(x_1,x_2,x_3) = (x_1,x_2,0)$ is linear .

(8) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Determine a linear map $T : V_3 \to V_2$ such that $A = (T : B_1, B_2)$, where $B_1 \& B_2$ are standard bases for $V_2 \& V_3$ respectively .

Que.3 (a) Let R^+ be the set of all positive real numbers .Define the operations as bellow : 5
$u+v = uv \;,\; \forall\; u,v \in R^+ \;;\; \alpha u = u^\alpha, \forall\; u \in R^+, \alpha \in \mathbb{R}$.
Prove that R^+ is a real vector space .

(b) A nonempty subset S of a vector space V is a subspace of V iff $\alpha u + \beta v \in S$, $\forall\; u\,,\; v \in S$ and for all scalar α , β . 3

OR

Que.3 (a) Show that $V_3 = \{(x_1,x_2,x_3) \;/\; x_1,x_2,x_3 \in \mathbb{R}\}$ is a real vector space under usual addition and scalar multiplication . 5

(b) Is the set $\{(x_1,x_2,x_3) \in V_3 / \; x_2 = \sqrt{2}x_1\}$ subspaces of V_3 ? Verify it . 3

Que.4 (a) Let S be a nonempty subset of a vector space V then prove that [S] is the smallest subspace of V containing S . 4

(b) In V_2 , show that (3 , 7) belongs to [(1,2),(0,1)] but does not belongs to [(1,2),(2,4)] . 4

OR

Que.4 (a) If $S = \{(1,-3,2),(2,-1,1)\}$ is a subset of V_3 , then show that $(1,7,-4) \in [S]$ but $(2,-5,4)$ does not belongs to [S] . 5

(b) If S is a nonempty subset of a vector space V then prove that [S] = S iff S is a subspace of V . 3

Que.5 (a) Is the set $S = \{(1,1,2),(-3,1,0),(1,-1,1),(1,2,-3)\}$ LD ? If set is LD then locate one of the vectors that belongs to the span of previous ones . Also find a LI subset A of S such that [A]=[S] . 5

(b) In any vector space V , prove that if a set is LI then any subset of it is also LI . 3

OR

Que.5 (a) Is the set $S = \{(1,1,2),(-3,1,0),(4,0,2),(1,-1,1)\}$ LD ? If set is LD then locate one of the vectors that belongs to the span of previous ones . Also find a LI subset A of S such that [A]=[S] . 5

(b) Let V be a vector space then prove that The set $\{v_1, v_2\}$ is LD iff $v_1 \& v_2$ are collinear . 3

Que.6 (a) If U and W are subspaces of a finite dimensional vector space V then prove that $dim(U+W) = dim(U) + dim(W) - dim(U \cap W)$. 4

(b) In a vector space V ,If $B = \{v_1, v_2, ..., v_n\}$ span V then prove that the following two conditions are equivalent 4
(i) B is LI
(ii) If $v \in V$, then the expression $v = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n$ is unique .

OR

Que.6 (a) Let the set $\{v_1, v_2, ..., v_k\}$ be a linearly independent subset of an n - dimensional vector space V . then prove that we can find vectors $\{v_{k+1}, v_{k+2}, ..., v_n\}$ such that the set $\{v_1, v_2, ..., v_k, v_{k+1}, ..., v_n\}$ is a basis for V . 5

(b) Is the subset $S = \{x-1, x^2+x-1, x^2-x+1\}$ from a basis for vector space P_2 ? Verify it . 3

Que.7 (a) If $T : U \to V$ is one - one and onto linear map , then prove that $\{T(u_1), T(u_2), ..., T(u_n)\}$ is LI set in V iff $\{u_1, u_2, ..., u_n\}$ is LI in U . 4

(b) Determine a linear map if $T : V_2 \to V_4$ defined by
$T(1,1) = (0,1,0,0)$, $T(1,-1) = (1,0,0,0)$. 4

OR

Que.7 (a) Let U and V be a vector space and $T : U \to V$ be any map then prove that T is linear iff $T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2)$, for all scalar α, β , $\forall\ u_1, u_2 \in U$. 4

(b) Determine a linear map if $T : V_2 \to V_4$ defined by
$T(1,1) = (1,1,1,1)$, $T(1,-1) = (-1,-1,-1,-1)$. 4

Que.8 (a) Let a linear map $T : V_2 \to V_2$ be defined by $T(x,y) = (x,-y)$. Find $(T : B_1, B_2)$, 8
where $B_1 = \{(1,1),(1,0)\}$; $B_2 = \{(2,3),(4,5)\}$

OR

Que.8 (a) Let a linear map $T : V_3 \to V_2$ be defined by $T(x,y,z) = (x+y, y+z)$. Find $(T : B_1, B_2)$, 8
where $B_1 = \{(1,1,2/3),(-1,2,-1),(2,3,1/2)\}$; $B_2 = \{(1,3),(1/2,1)\}$

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