

[80/A-13]

SEAT No. _____

SARDAR PATEL UNIVERSITY

B.SC. SEM-I (NC) EXAMINATION

30th October 2018, Tuesday

02.00 p.m. to 04.00 pm

US01EMTH02

(Mathematics)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Principle diagonal entries of skew-symmetric matrix are

- (a) Real (b) Complex (c) Zero (d) None

(2) Inverse of $(0, -1) = \dots \dots \dots$

- (a) $(0, 1)$ (b) $(1, 0)$ (c) $(-1, 0)$ (d) $(0, 0)$

(3) Conjugate of $z = -8 + 2i$ is

- (a) $-8 - 2i$ (b) $-4 + i$ (c) $8 + 2i$ (d) $8 - 2i$

(4) Range of the constant function is

- (a) Empty set (b) singleton set (c) \mathbb{Z} (d) \mathbb{N}

(5) If $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x^2$ and $g : \mathbb{N} \rightarrow \mathbb{N}$ defined by $g(x) = x^3$ then $fog(x) = \dots \dots \dots$

- (a) x^5 (b) x^4 (c) x^6 (d) None

(6) Measure of angle between \vec{i} and \vec{j} is

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{3}$

(7) If $\Delta = 0$ then quadratic equation has

- (a) Two complex root (b) Two equal root (c) Two real root (d) none

(8) $|(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})| = \dots \dots \dots$

- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

(9) If $x = \log_5(125)$ then $x = \dots \dots \dots$

- (a) 4 (b) 2 (c) 3 (d) 5

(10) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \dots \dots \dots$

- (a) 4 (b) -1 (c) 0 (d) 1

Q.2 Attempt any ten in short: [20]

(1) Find value of $\sin 150^\circ$ and $\tan \left(\frac{3\pi}{4}\right)$.

(2) Define Transpose of a matrix.

(3) Evaluate $(2, 3, 1) \times (1, 2, 3)$.

(PTE)

- (4) Express $2^7 = 128$ and $8^0 = 1$ in Logarithmic form.
- (5) Find range of the function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x - [x]$.
- (6) Define vector and unit vector.
- (7) Solve : $2x + 3y = 7$; $4x - y = 9$.
- (8) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is one one ?
- (9) Find $\alpha \in \mathbb{C}$ which satisfy $(5, 6) + \alpha = (2, -1)$.
- (10) If $\bar{x} = (1, 1, 1)$, $\bar{y} = (1, 0, 0)$ then find $\bar{x} - \bar{y}$.
- (11) Express $i^9 + i^{10} + i^{11} - 3i^{12}$ in $a + ib$ form.

(12) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ then find $A + A^T$. Is it symmetric.

Q.3

- (a) If $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x^2$ and $g : \mathbb{N} \rightarrow \mathbb{N}$ defined by $g(x) = x^3$ then find fog and gof. [5]
- (b) Solve: (i) $3\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) + 26 = 0$
(ii) $\sqrt{4x+1} + \sqrt{x+1} = 3$ [5]

OR

Q.3

- (c) Find conjugate and Modulus of following : [5]
 - (i) $(2 + 7i)^3$
 - (ii) $\frac{(8 - 3i)(6 - i)}{2 - 2i}$
- (d) Define one-one and onto function. Check which of the following function are one - one and onto? [5]
 - (i) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 4x + 5$
 - (ii) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + |x|$.

Q.4

- (a) Prove that $\left(\frac{1 - \tan \theta}{1 - \cot \theta}\right)^2 = \frac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$. Also, find the value of $\tan\left(\frac{-17\pi}{4}\right)$. [5]
- (b) Simplify the following : [5]
 - (i) $7\log(8/5) - 6\log(4/15) + 3\log(5/72)$
 - (ii) $\log(a^2/bc) + \log(b^2/ca) + \log(c^2/ab)$.

OR

Q.4

- (c) If $\sin \theta + \operatorname{cosec} \theta = 2$ then prove that $\sin^n \theta + \operatorname{cosec}^n \theta = 2$, $n \in \mathbb{N}$. [5]
- (d) Solve the following : [5]
 - (i) $x^{\log_9 x} = 81x$
 - (ii) $\log_x 4 + \log_x 16 + \log_x 64 = 12$

Q.5

(a) Solve $3x + 2y = 13xy$; $-2x + 5y = 4xy$ by using Cremer's rule. [5]

(b) If $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ [5]
then prove that $A(B + C) = AB + AC$.

OR

Q.5

(c) Prove that

$$\begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = -(x-y)(y-z)(z-x).$$

[5]

(d) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then prove that $A^{-1} = \frac{1}{19}A$. [5]

Q.6

(a) If $\bar{x} = (1, 0, 0)$, $\bar{y} = (0, 1, 1)$, $\bar{z} = (1, 1, 1)$ then find $[\bar{x}, \bar{y}, \bar{z}]$. [5]

(b) Define vector and unit vector. Find direction cosines of the
 $(1,1,1)$; $(0,1,1)$; $(2,2,1)$; $\bar{i} + \bar{j}$; $3\bar{i} + 4\bar{j} - 2\bar{k}$. [5]

OR

Q.6

(c) If $(2a, a, -4)$ and $(a, -2, 1)$ are orthogonal then find a . [5]

(d) Solve: $5x + 8y + z = 2$; $2y + z = -1$; $4x + 3y - z = 3$. [5]

— X —

where $\rho_{\alpha\beta} = \rho_{\alpha\beta}(t, t_0)$, $\rho_{\alpha\beta}^0 = \rho_{\alpha\beta}(t_0, t_0)$, and $\rho_{\alpha\beta}^{(0)} = \rho_{\alpha\beta}(t_0, t_0)$.

It follows from (2.1) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(0)}$ and $\rho_{\alpha\beta}^{(0)} = \rho_{\alpha\beta}^{(0)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(0)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(1)}(t', t_0) dt' \quad (2.2)$$

where $\rho_{\alpha\beta}^{(1)} = \rho_{\alpha\beta}^{(1)}(t, t_0)$ and $\rho_{\alpha\beta}^{(1)} = \rho_{\alpha\beta}^{(1)}(t, t_0)$.

It follows from (2.2) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(1)}$ and $\rho_{\alpha\beta}^{(1)} = \rho_{\alpha\beta}^{(1)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(1)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(2)}(t', t_0) dt' \quad (2.3)$$

where $\rho_{\alpha\beta}^{(2)} = \rho_{\alpha\beta}^{(2)}(t, t_0)$ and $\rho_{\alpha\beta}^{(2)} = \rho_{\alpha\beta}^{(2)}(t, t_0)$.

It follows from (2.3) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(2)}$ and $\rho_{\alpha\beta}^{(2)} = \rho_{\alpha\beta}^{(2)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(2)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(3)}(t', t_0) dt' \quad (2.4)$$

where $\rho_{\alpha\beta}^{(3)} = \rho_{\alpha\beta}^{(3)}(t, t_0)$ and $\rho_{\alpha\beta}^{(3)} = \rho_{\alpha\beta}^{(3)}(t, t_0)$.

It follows from (2.4) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(3)}$ and $\rho_{\alpha\beta}^{(3)} = \rho_{\alpha\beta}^{(3)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(3)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(4)}(t', t_0) dt' \quad (2.5)$$

where $\rho_{\alpha\beta}^{(4)} = \rho_{\alpha\beta}^{(4)}(t, t_0)$ and $\rho_{\alpha\beta}^{(4)} = \rho_{\alpha\beta}^{(4)}(t, t_0)$.

It follows from (2.5) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(4)}$ and $\rho_{\alpha\beta}^{(4)} = \rho_{\alpha\beta}^{(4)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(4)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(5)}(t', t_0) dt' \quad (2.6)$$

where $\rho_{\alpha\beta}^{(5)} = \rho_{\alpha\beta}^{(5)}(t, t_0)$ and $\rho_{\alpha\beta}^{(5)} = \rho_{\alpha\beta}^{(5)}(t, t_0)$.

It follows from (2.6) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(5)}$ and $\rho_{\alpha\beta}^{(5)} = \rho_{\alpha\beta}^{(5)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(5)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(6)}(t', t_0) dt' \quad (2.7)$$

where $\rho_{\alpha\beta}^{(6)} = \rho_{\alpha\beta}^{(6)}(t, t_0)$ and $\rho_{\alpha\beta}^{(6)} = \rho_{\alpha\beta}^{(6)}(t, t_0)$.

It follows from (2.7) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(6)}$ and $\rho_{\alpha\beta}^{(6)} = \rho_{\alpha\beta}^{(6)}(t, t_0)$.

Let us now consider the case where $\rho_{\alpha\beta}^{(6)} = 0$. Then we have

$$\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \int_{t_0}^t \rho_{\alpha\beta}^{(7)}(t', t_0) dt' \quad (2.8)$$

where $\rho_{\alpha\beta}^{(7)} = \rho_{\alpha\beta}^{(7)}(t, t_0)$ and $\rho_{\alpha\beta}^{(7)} = \rho_{\alpha\beta}^{(7)}(t, t_0)$.

It follows from (2.8) that $\rho_{\alpha\beta} = \rho_{\alpha\beta}^0 + \rho_{\alpha\beta}^{(7)}$ and $\rho_{\alpha\beta}^{(7)} = \rho_{\alpha\beta}^{(7)}(t, t_0)$.