Seat No.: .....

No. of Printed Pages : 3

[15 & A 17]
(Eng.)

SARDAR PATEL UNIVERSITY

B.SC. SEM-I EXAMINATION

23rd November 2016 , Wednesday

10.00 a.m. to 12.00 noon

US01EMTH02

(Mathematics)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Principle diagonal entries of skew-symmetric matrix are .....

(a) Real (b) Complex (c) Zero (d) None of these

(2) If $z = 3 - 4i$ then $|z| =$ .....

(a) 4 (b) 3 (c) 5 (d) 25

(3) Value of $i^{17} =$ .....

(a) 1 (b) i (c) $-i$ (d) -1

(4) If $\overline{x} = (1,1,1)$, $\overline{y} = (1,1,0)$, $\overline{z} = (1,0,0)$ then $[\overline{x}\ \overline{y}\ \overline{z}] =$ .....

(a) 1 (b) -1 (c) 0 (d) 2

(5) Exponential form of $\log_{10} 1000 = 3$ is. .....

(a) $3^{10} = 1000$ (b) $10^3 = 1000$ (c) $1000^{10} = 3$ (d) $10^{1000} = 3$

(6) If $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = x^2$ and $g : \mathbb{N} \to \mathbb{N}$ defined by $g(x) = x^3$ then fog(x)=. .....

(a) x^5 (b) x^4 (c) x^6 (d) None of these

(7) Measure of angle between $\overline{i}$ and $\overline{j}$ is. .....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{3}$

(8) If $\frac{-b \pm \sqrt{\Delta}}{2a}$ is solution of quadratic equation than $\Delta =$ .....

(a) $c^2 - 4ac$ (b) $a^2 - 4bc$ (c) $b^2 - 4ac$ (d) 0

(9) If $x = \log_5(125)$ then $x =$ .....

(a) 4 (b) 2 (c) 3 (d) 5

(10) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} =$ .....

(a) 4 (b) -1 (c) 0 (d) 1

Q.2 Attempt any **ten** in short: [20]

(1) If $A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 6 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 6 & 3 \\ 0 & 2 & 7 \end{bmatrix}$ then find $A + B$ and $A - B$.

(2) Define Determinant of order 3×3 .

(3) Express $i + i^5 + i^9 + 5i^{13}$ in $a + ib$ form.

(4) Express $2^7 = 128$ and $8^0 = 1$ in Logarithmic form.

(5) Find range of the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 4x$.

(6) Define vector and unit vector.

(7) Evaluate $(2, 3, 1) \times (1, 2, 3)$.

(8) Define : Quadractic equation.

(9) Prove that

$$\begin{vmatrix} 0 & -x & -y \\ x & 0 & z \\ y & z & 0 \end{vmatrix} = 0$$

(10) If $\overline{x} = (2, 0, 1)$, $\overline{y} = (1, 2, 4)$ then find $5\overline{x} + 3\overline{y}$.

(11) Find value of sin 150^0 and tan $\left(\frac{3\pi}{4}\right)$.

(12) Find $\alpha \in \mathbb{C}$ which satisfy $(5, 6) + \alpha = (2, -1)$.

Q.3

(a) If $f : \mathbb{R} \to \mathbb{R}$ defined by [5]
$f(x) = x^2 + x + 1$, $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x - 1$ and
$h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 2x$ then find fog , foh , fo(goh) , (fog)og , gog.

(b) Solve: (i) $3\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) + 26 = 0$

(ii) $\sqrt{4x+1} + \sqrt{x+1} = 3$ [5]

OR

Q.3

(c) Find conjugate and Modulus of following : [5]

(i) $(2 + 7i)^2$ (ii) $\frac{1-i}{1+i}$

(d) Define one-one and onto function. Check which of the following [5]
function are one - one and onto?

(i) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 4x + 5$

(ii) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.

Q.4

(a) Prove that $\sin\left(\frac{10\pi}{3}\right)\cos\left(\frac{11\pi}{6}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{5\pi}{6}\right) = 1$ [5]

(b) Solve the following :

(i) $\log_x 4 + \log_x 16 + \log_x 64 = 12$ (ii) $\log x - \log(x-1) = \log 3$ [5]

OR

Q.4

(c) Prove that $\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \frac{1+\tan^2\theta}{1+\cot^2\theta}$. Also, find the value of $\tan\left(\frac{-17\pi}{4}\right)$. [5]

(d) Simplify the following :

(i) $\log(11/15) + \log(490/297) - 2\log(7/9)$

(ii) $\log_b a \times \log_c b \times \log_a c$. [5]

Q.5

(a) Solve $2x + 3y - 8 = 0$; $5x - 4y + 3 = 0$ by using Cremer's rule. [5]

(b) If $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ then prove that $A(B+C) = AB + AC$. [5]

OR

Q.5

(c) Prove that

$$\begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = -(x-y)(y-z)(z-x).$$

[5]

(d) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then prove that $A^{-1} = \frac{1}{19}A$. [5]

Q.6

(a) If the measure of the angle between $\bar{i}+\sqrt{3}\,\bar{j}$ and $\sqrt{3}\,\bar{i}+a\bar{j}$ is $\pi/3$ then find a. [5]

(b) Solve: $5x + 8y + z = 2; 2y + z = -1; 4x + 3y - z = 3$. [5]

OR

Q.6

(c) If $\bar{x} = (1,1,2)$, $\bar{y} = (1,2,1)$, $\bar{z} = (2,1,1)$ then find $\bar{x} \times (\bar{y} \times \bar{z})$. [5]

(d) Define dot and Cross Product. Find direction cosines of the [5]
(1,1,1) ; (0,1,1) ; (2,2,1) ; $\bar{i}+\bar{j}$; $3\bar{i}+4\bar{j}-2\bar{k}$.

X=X=X