[25/A-16/Eng] Seat No :______ No. of printed pages : 3

SARDAR PATEL UNIVERSITY
B Sc 1st Semester Examination
2016
Wednesday, 16th April
10.00 am to 12.00 noon
US01CMTH02 – MATHEMATICS
Calculus and Differential Equations

Maximum Marks : 70

1. Answer the following questions by selecting the correct choice from the given option : [10]

(1) If $y = (3x-7)^{100}$ then $y_{101} =$ _______

a. $(100)!\ 3^{100}(3x-7)$ b. $(101)!\ 3^{101}(3x-7)$

c. $(100)!\ 3^{100}$ d. 0

(2) If $y = e^x cosx$ then $y_n =$ ________

a. $(\sqrt{2})^n e^x cos(x + n\frac{\pi}{4})$ b. $(2^n)e^x cos(x + n\frac{\pi}{4})$

c. $(2^n)e^x sin(x + n\frac{\pi}{4})$ d. $(\sqrt{2})^n e^x sin(x + n\frac{\pi}{4})$

(3) If $r = a\, e^{b\theta}$ then $tan\emptyset =$ _______

a. $\frac{1}{b}$ b. b

c. $-\frac{1}{b}$ d. a

(4) Radius of Curvature of the circle $x^2 + y^2 = a^2$ is _______.

a. $\frac{1}{a}$ b. $-\frac{1}{a}$

c. a d. $-a$

(5) Solution of $p^2 - x^2 = 0$ is $y =$ _______.

a. $(y-x-c)(y+x-c) = 0.$ b. $(y-x^2-c)(y+x^2-c) = 0.$

c. $(y-x-c) = 0.$ d. $(y-x^2-c) = 0.$

(6) If $y = \frac{1}{x}$ then $y_n =$ ________

a. $\frac{(-1)^n\, n!}{x^n}$ b. $\frac{(-1)^n\, n!}{x^{n-1}}$

c. $\frac{(-1)^n\, n!}{x^{n+1}}$ d. $\frac{(-1)^{n-1}\,(n-1)!}{x^n}$

(7) If $u = sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} =$ ____________

a. $tanu$ b. $2tanu$

c. $2sinu$ d. $sinu$

(P.T.O.)

(8) If $f(x,y) = \frac{x-y}{x+y}$ then $f_x =$ _______

a. $\frac{2x}{(x+y)^2}$

b. $\frac{-2x}{(x+y)^2}$

c. $\frac{-2y}{(x+y)^2}$

d. $\frac{2y}{(x+y)^2}$

(9) The general solution of the differential equation $y = px + e^p$, $p = \frac{dy}{dx}$ is _______

a. $y = cx + c$

b. $y = cx + e^c$

c. $x = cy - c$

d. $x = cy + c^2$

(10) ________ has infinite radius of curvature at any point

a. Circle with radius 1

b. Parabola $y^2 = 4x$

c. Line $y = x$

d. None

2. Answer any TEN of the following. [20]

1) If $y = sin^2(2x)$, find y_n
2) If $y = \frac{1}{4x^2-9}$, find y_n
3) If $r = a(1 + \cos\theta)$, the find ϕ
4) Find the radius of curvature at any point on the curve $s = 8asin^2\left(\frac{\Psi}{6}\right)$
5) Find $\frac{ds}{dx}$ for the curve $x = a(t - sint)$, $y = a(1 - cost)$
6) Find the length of curve $y = coshx$ measured from $(0,1)$ to $(1,e)$
7) If $z = sin^{-1}(3t - 4t^3)$ then find $\frac{dz}{dt}$.
8) If $u = e^x(xcosy - ysiny)$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
9) Find $\frac{dy}{dx}$ if $e^x + e^y = 2xy$
10) Solve : $(ax + hy + g)dx + (hx + by + f)dy = 0$
11) Solve : $(p^2 - 12p + 35)(p - x) = 0$, $p = \frac{dy}{dx}$
12) Solve : $y = pxlogx$, $p = \frac{dy}{dx}$

3. If $= \left(x + \sqrt{x^2+1}\right)^m$, then prove that $(1 + x^2)y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$,
Also find $y_n(0)$ [10]

OR

3. (a) If $y = e^{ax}\, sin(bx + c)$ then prove that $y_n = r^n e^{ax}\, sin(bx + c + n\emptyset)$ where

$a = rcos\emptyset$, $b = r\, sin\emptyset$ [5]

(b) If $y = log(ax + b)$ then prove that $y_n = \frac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n}$ [5]

4. Define radius of curvature. Prove that the radius of curvature for the curve $r = f(\theta)$ is given by

$\rho = \frac{(r^2+r_1^2)^{\frac{3}{2}}}{r^2+2\,r_1^2-r\,r_2}$ where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$ [10]

OR

4. Show that he entire length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$ [10]

5. (a) State and prove Euler's theorem [5]

(b) Verify Euler's theorem for $z = x^n \log\left(\frac{y}{x}\right)$ [5]

OR

5. (a) If $z = f(x, y)$, $x = r\cos\theta,\ y = r\sin\theta$ then prove that $\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2 = \left[\frac{\partial z}{\partial r}\right]^2 + \frac{1}{r^2}\left[\frac{\partial z}{\partial \theta}\right]^2$ [5]

(b) If $H = f(2x - 3y,\ 3y - 4z,\ 4z - 2x)$, then prove that $\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0$ [5]

6. Write Clairaut's differential equation and explain method for solving it. [10]

Hence solve the differential equation $\sin px \cos y = \cos px \sin y - \log p + p^3$, where $p = \frac{dy}{dx}$

OR

6. (a) Solve the differential equation $p^2 + 2py\cot x = y^2$ where $p = \frac{dy}{dx}$ [5]

(b) Find the orthogonal trajectories of the semi-cubical parabolas $a\,y^2 = x^3$ [5]

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