[21 & A-11] Seat No. __________ No. of printed pages: 03

SARDAR PATEL UNIVERSITY
B. Sc. (I Semester) Examination
2016
Tuesday, 15th November
10.00 am - 12.00 pm
US01CMTH01 - Mathematics
Analytical Geometry & Complex Numbers

Total Marks : 70

Note : Figures to the right indicates full marks of question.

Q-1 Answer the following by selecting the correct answer from the given options: (10)

(1) Asymptotes of $y = \dfrac{x^2-1}{x^2-4}$ are ....

(a) x = 2 , - 2 ; y = 1 (b) x = 1 , - 1 ; y = 1 (c) x = 2 , - 2 ; y = 0 (d) x = 1 , - 1 ; y = -2.

(2) $y = \dfrac{2}{3x}$ is symmetric about ....

(a) x-axis (b) y - axis (c) origin (d) none of these .

(3) The shape of lemniscates looks, like..........

(a) 8 (b) flower (c) rose (d) heart shape .

(4) The curve of $r = sin4\theta$ is symmetric about ....

(a) polar axis (b) normal axis (c) pole (d) polar axis , normal axis and pole .

(5) Polar equation of vertical line left to the pole is ....

(a) $p = r\cos\theta$ (b) $p = r\sin\theta$ (c) $p = -r\sin\theta$ (d) $p = -rcos\theta$.

(6) If eccentricity $e < 1$ then conic is ....

(a) hyperbola (b) parabola (c) circle (d) ellipse .

(7) Centre of the circle $r = -9\sin\theta$ is ....

(a) $(3, 3\pi/2)$ (b) $(9/2, 3\pi/2)$ (c) $(9/2, \pi/2)$ (d) $(9, 3\pi/2)$.

(8) Amplitude of $-\sqrt{3}+i$ is ....

(a) 60^o (b) 150^o (c) 30^o (d) 120^o .

(9) Cube roots of unity are ....

(a) $1, -1$ (b) $1, -\dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}$ (c) $1, \pm\dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}$ (d) $1, \dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}$.

(10) The value of $z + \frac{1}{z} =$.........

(a) $2cos\theta$ (b) $cos2\theta$ (c) $2sin\theta$ (d) 0 .

(P.T.O.)

Q-2 Answer any ten of the following: (20)

(1) Find the parametric equation of $\sqrt{x}+\sqrt{y}=\sqrt{a}$.
(2) Discuss symmetries of the curve $y=\frac{x^3}{x^2-1}$.
(3) Transfer the equation in cartesian form: $r = tan\theta sec\theta$.
(4) Express the point $(-\sqrt{3},1)$ in polar form.
(5) Identify the curve $3(1+cos\theta)$.
(6) find the radius and center of a circle $r=5cos\theta$.
(7) find the polar equation of conic if directrix passes through a point $(5,\pi/2)$ and $e=2/3$.
(8) Find polar equation of circle centre at $(5,225^0)$ and radius is 2.
(9) Find x-intercept and y-itercept of a curve $y=\frac{2}{x^2-x-2}$.
(10) Find the real and imaginary part of $z=(1+2i)(1-3i)$
(11) simplify $(cos\theta - isin\theta)^n$
(12) If $z=x+iy$ the find $\frac{z+\bar{z}}{2}$

Q-3(a) If a curve is given by $x=f(t); y=g(t)$ and that both x and y get numerically large as t approaches some number,say a.Then prove that oblique asymptote to the curve ,if it exist, is given by $y=mx+c$, where
$m=\lim_{t\to a}\frac{dy}{dx}$ and $c=\lim_{t\to a}(y-mx)$. [5]

Q-3(b) Sketch the curve given by $y=x^3-3x^2+2x$. [5]

OR

Q-3(a) Obtain the parametric equation of cycloid. [5]

Q-3(b) Find the asymptotes for the given curve by $x=t+\frac{1}{t^2}, y=t-\frac{1}{t^2}$ [5]

Q-4(a) State when a polar curve is symmetric with respect to normal axis? Prove it. [5]

Q-4(b) Sketch the curve $r=2+cos\theta$. [5]

OR

Q-4(a) State when a polar curve is symmetric with respect to polar axis? Prove it. [5]

Q-4(b) Sketch the curve $r^2=-16sin2\theta$ [5]

Q-5(a) In usual notation prove that the polar equation of conic is,
$r=\frac{p\,e}{1\pm e\,cos\theta}$. [5]

Q-5(b) Define: Reciprocal curve.
Identify curve $r=1+2cos\theta$ also find its reciprocal curve sketch both of the curve with the same frame of reference. [5]

OR

Q-5(a) Prove that polar equation of circle with centre (r_1,θ_1) and radius a is given by $r^2+r_1^2-2rr_1cos(\theta-\theta_1)=a^2$.Also find equation of circle if center is on polar axis. [5]

Q-5(b) In usual notation prove that polar equation of line is $p = r\cos(\theta - \omega)$.
Hence obtain equation of line
(i) perpendicular to polar axis (ii)parallel to polar axis. [5]

Q-6(a) Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$. [5]

Q-6(b) Solve $x^4 - x^3 + x^2 - x + 1 = 0$ by using De Moiver's theorem. [5]

OR

Q-6(a) State and prove De-Moiver's theorem. [5]

Q-6(b) Expand $\cos^8\theta$ in a series of cosines. [5]

— X —

(3)