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SARDAR PATEL UNIVERSITY

BSc (I Sem.) Examination

Friday, 15 November 2013

2.30 - 4.30 pm

US01CMTH02 - Calculus and Differential Equations

Total Marks: 70

Note: Figures to the right indicate full marks of the questions.

Q.1 Answer the following questions by selecting the correct choice from the given options. [10]

(1) $\frac{d^n}{dx^n}(x^n) =$ ____________.

(a) (2n)! (b) 0 (c) 1 (d) n!

(2) If $y=e^{4x}+e^{2x}$ then $y_n =$ ____________.

(a) $e^{2x}(2^n e^{2x}+1)$ (b) $2^n e^{2x}(2^n e^{2x}+1)$

(c) $e^{2x}(2^n e^{2x}-1)$ (d) None

(3) If y=sin3x then $y_{10}=$ ____________.

(a) $3^{10}\sin 3x$ (b) $3^{10}\cos 3x$

(c) $-3^{10}\sin 3x$ (d) $-3^{10}\cos 3x$

(4) $\frac{ds}{dt} =$ ____________.

(a) $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ (b) $\sqrt{1+\left(\frac{dx}{dt}\right)^2}$

(c) $\sqrt{1-\left(\frac{dy}{dt}\right)^2}$ (d) None

(5) For a curve $r=f(\theta)$, $\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} =$ ____________.

(a) $\frac{df}{d\theta}$ (b) $\frac{dr}{d\theta}$ (c) $\frac{ds}{d\theta}$ (d) $\frac{d\theta}{ds}$

(6) The reciprocal of curvature at a point is known as ____________ at the point.

(a) radius of curve (b) radius of curvature

(c) rate of bending (d) None

(7) The degree of homogeneous function $f(x,y)=\frac{x^2+y^2}{x-y}$ is __________.

(a) 0 (b) 3 (c) 2 (d) 1

(8) Partial derivative of $z=f(x,y)$ w. r. t x is denoted by ____________.

(a) $\frac{\partial f}{\partial x}$ (b) $\frac{df}{d\theta}$ (c) $\frac{dz}{dx}$ (d) None

(9) A differential equation of the form $y=px+f(p)$ is known as __________ equation.

(a) Euler's (b) Clairut's (c) Exact (d) None

(10) $(x^2-2xy-y^2)dx-(x+y)^2dy=0$ is ____________ differential equation.
(a) Clairut's (b) not exact (c) exact (d) None

Q.2 Answer the following questions in short. **(Any Ten)** [20]

(1) Find the angle between radius vector and tangent to the curve $r=a(1-\cos\theta)$

(2) If $y=\cos 3x$ then find y_4

(3) Find y_n if $y=x\sin x$

(4) For the curve $y=a\sin 2x$, find $\frac{ds}{dx}$

(5) Find radius of curvature at any point on the curve $S=8a\sin^2\frac{\psi}{6}$

(6) Find ρ for the curve $r=a(1+\cos\theta)$

(7) If $u=\frac{x^3+y^3}{xy}$ then find $x\cdot\frac{\partial u}{\partial x}+y\cdot\frac{\partial u}{\partial y}$

(8) Verify Euler's theorem for the function $z=x^2y-xy^2$

(9) For $u=x^3-3xy^2$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(10) Check whether the differential equation $xdx+ydy+\frac{xdy-ydx}{x^2+y^2}=0$ is Exact or not.

(11) Solve: $y=px+\frac{3}{p}$.

(12) Solve: $y^2-2pxy+p^2(x^2-1)=m^2$

Q.3
(a) State and prove Leibniz theorem. [05]
(b) If $y=e^{ax}\sin(bx+c)$ then find y_n. [05]

OR

Q.3
(a) In usual notations prove that $\tan\theta=\frac{r}{\frac{dr}{d\theta}}$ [05]

(b) If $y=(ax+b)^m$ with $m\in\mathbb{N}$ then prove that $y_n=\frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$ [05]

Q.4
(a) Find the entire length of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ [05]

(b) In usual notations prove that $\rho=\frac{(1+y_1^2)^{3/2}}{y_2}$ [05]

OR

Q.4

(a) Show that the intrinsic equation of the curve $y^3 = ax^2$ is $27s = 8a(\sec^3\psi - 1)$ [05]

(b) For a polar equation $r = f(\theta)$ prove that $\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ [05]

Q.5

(a) If $H = f(2x-3y, 3y-4z, 4z-2x)$ then prove that $\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0$ [05]

(b) State and prove Euler's theorem for the function z = f(x, y) [05]

OR

Q.5

(a) If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then prove that $x\cdot\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y} = 3\tan u$ [05]

(b) If A, B, C are angles of a ΔABC such that $\sin^2 A + \sin^2 B + \sin^2 C = K$, K is constant. Prove that $\frac{dB}{dC} = \frac{\tan C - \tan A}{\tan A - \tan B}$ [05]

Q.6 Prove that the necessary and sufficient condition for the differential equation Mdx+Ndy=0 to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ [10]

OR

Q.6 Solve: $p^2 + 2py\cot x = y^2$ [10]

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