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SARDAR PATEL UNIVERSITY

BSc (First Semester) Examination

2013

Tuesday, 12th November

2.30 to 4.30 pm

US01CMTH01 - Mathematics

Analytic Geometry and Complex Numbers

Total Marks: 70

Q.1 Answer the following by selecting correct choice from the given options. (10)

(1) ________ is a closed curve.
(a) Parabola (b) Hyperbola (c) Ellipse (d) Helix

(2) The curve of $y = \frac{2}{(x+1)(x-2)}$ has ________ branches.
(a) 1 (b) 3 (c) 2 (d) 4

(3) The curve xy= 25 is symmetric with respect to ________.
(a) Origin (b) x-axis (c) y-axis (d) None

(4) The perpendicular distance of line $2\sqrt{2} = r(\sqrt{3}\cos\theta + \sin\theta)$ from the pole is ________.
(a) 1 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{2}$

(5) The curve $r = 2\text{-}4\cos\theta+\sin\theta$ represnets ________.
(a) Spiral (b) rose curve (c) limacon (d) parabola

(6) The curve $r = 3 + 2\sin\theta$ is symmetric with respect to ________
(a) normal axis (b) polar axis (c) pole (d) None

(7) if eccentricity e>1 then conic is ________
(a) parabola (b) circle (c) hyperbola (d) ellipse

(8) The centre of a circle $r = 6\sin\theta$ is ________
(a) $(-3,\frac{\pi}{2})$ (b) $(3,\frac{\pi}{2})$ (c) $(-3,\pi)$ (d) $(3, \pi)$

(9) $(\text{cis}\,\theta)^{\frac{1}{q}}$ has only ________ distinct values.
(a) $\frac{1}{q}$ (b) 1 (c) 2q (d) q

(10) Cube root of unity are ________
(a) 1, -1 (b) $1, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ (c) $1, \pm\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ (d) $1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

Q.2 Answer the following in short (Attempt **Any Ten**) (20)

(1) Find parametric equation for $\sqrt{x} + \sqrt{y} = \sqrt{a}$

(2) Discuss all symmetries of $y = \frac{4-x^2}{x^2-9}$

(3) Find equation of tangent to the curve given by $x= a\cos\theta$, $y=b\sin\theta$

(4) Transfer the equation $r=\tan\theta+\sec\theta$ into Cartesian form.

(5) Express the point (3,40°) in three other ways such that $-2\pi \le \theta \le 2\pi$

(6) Express the point $(\sqrt{3},1)$ in polar form.

(7) Write down polar equation of horizontal line through the point (3, 90°).

(8) Find radius and centre of $r = 3\sin\theta$

(9) Find polar equation of circle whose centre is at (7, 60°) and radius is 10.

(10) If $2\cos\theta = x + \frac{1}{x}$, then prove that $2\cos r\theta = x^r + \frac{1}{x^r}$

(11) Find $z + \frac{1}{z}$, where z = 4 +5i

(12) Express the complex number $z = -\sqrt{3}+i$ in modulus-amplitude form.

Q.3

(a) If a curve is given by x=f(t), y=g(t) and both x and y gets numerically large as t approaches to some number say 'a' then prove that the oblique asymptote to the curve if exist is given by y=mx+c where $m = \lim_{t \to a} \frac{dy}{dx}$, $c = \lim_{t \to a} (y-mx)$ (05)

(b) Discuss intercepts, symmetries, asymptotes, sign of the function and hence sketch the curve given by xy-y-2x =0 (05)

OR

Q.3

(a) Find asymptotes to the curve given by $x=t + \frac{1}{t^2}$; $y= t - \frac{1}{t^2}$ (05)

(b) Obtain parametric equations of cycloid. (05)

Q.4

(a) When a curve given by polar equation is symmetric with respect to normal axis ?Justify your answer. (05)

(b) Sketch the curve $r = 2 + 3\cos\theta$ (05)

OR

Q.4

(a) When a curve given by polar equation is symmetric with respect to polar axis ? Justify your answer. (05)

(b) Sketch the curve $r = 5 \sin 3\theta$ (05)

Q.5 In usual notations prove that (10)

$$r = \frac{pe}{1 \pm e\cos\theta}$$

OR

Q.5 Identify and describe the curve $r=3(1+\cos\theta)$ and its reciprocal curve. Hence sketch them in the same frame of reference. (10)

Q.6

(a) State and prove De Moivres therem for complex number. (05)

(b) Prove that, (05)

$$\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1$$

OR

Q.6

(a) Prove that (05)

$$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$$

(b) Solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$ by using De Moivres theorem. (05)