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SARDAR PATEL UNIVERSITY
BSc. (I SEM.) (CBCS) EXAMINATION
Tuesday, 27th November 2012
2.30 pm - 4.30 pm
US01CMTH02 : Mathematics
(Calculus and Differential Equations)

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Answer the following questions by selecting the most appropriate option. Write down the option in your answer book. [10]

(1) If $y = 7^{5x}$ then $y_n =$______.

(a) $5^n 7^{5x}$ (b) $7^n (\log 5)^n 7^{5x}$

(c) $7^n \cdot 7^{5x}$ (d) $5^n (\log 7)^n \cdot 7^{5x}$

(2) If y=e^x then y_{16}=______.

(a) 0 (b) e^x

(c) 1 (d) e^{-x}

(3) If y=cos3x then y_n=______.

(a) $3^n \cos(3x + \frac{n\pi}{2})$ (b) $3^n \cos(3x + \frac{\pi}{2})$

(c) $3^n \sin(3x + \frac{n\pi}{2})$ (d) $3^n \sin(3x + \frac{\pi}{2})$

(4) $\sqrt{1 + (\frac{dy}{dx})^2}$ =________.

(a) ρ (b) $\frac{1}{\rho}$

(c) $\frac{ds}{dx}$ (d) $\frac{ds}{dy}$

(5) For a polar curve, ρ =________.

(a) $\frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$ (b) $\frac{(r^2 + r_2^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

(c) $\frac{(1 + r_1^2)^{3/2}}{r_2}$ (d) $\frac{(1 + r_2^2)^{3/2}}{r_1}$

(6) $\frac{\partial}{\partial x}(\frac{\partial z}{\partial y}) =$________.

(a) $\frac{\partial z}{\partial y}$ (b) 0

(c) $\frac{\partial^2 x}{\partial y \partial x}$ (d) $\frac{\partial^2 z}{\partial x \partial y}$

(7) For a function y of x implicitly described by f(x, y) = c, $\frac{dx}{dy}$ =________.

(a) $\frac{fx}{fy}$ (b) $\frac{fy}{fx}$

(c) $-\frac{fx}{fy}$ (d) $-\frac{fy}{fx}$

(8) In usual notations, $\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=$________.

(a) $\frac{dz}{dx}$ (b) $\frac{dz}{dt}$

(c) $\frac{dz}{dy}$ (d) $\frac{dy}{dx}$

(9) The notation p=_______.

(a) $\frac{\partial y}{\partial x}$ (b) $\frac{\partial x}{\partial y}$

(c) $\frac{dy}{dx}$ (d) $\frac{dx}{dy}$

(10) The general solution of the differential equation $y = px + \frac{5}{p}$ is ______.

(a) $y = x + 5$ (b) $y = cx + \frac{5}{c}$

(c) $cx + \frac{5}{c} = 0$ (d) $y = cp + \frac{5}{c}$

Q.2 Write down any answer of **Any Ten** questions in short. [20]

(1) If $y=e^{mx}$, then prove that $y_n= m^n e^{mx}$.

(2) If y=sin(ax+b) then find y_n.

(3) Find ϕ for the curve r=a(1+cosθ).

(4) Find ρ for r=aθ.

(5) Find $\frac{ds}{dx}$ for $y = a\cosh + \frac{x}{a}$

(6) Find the point of intersection of r=a(1+cosθ) and r=-acosθ.

(7) State theorem on total differential.

(8) Define: Homogeneous function

(9) State Euler's theorem for function of two variables.

(10) Examine whether $(x^2 - 2xy - y^2)dx - (x+y)^2\,dy = 0$ is exact or not.

(11) Define: Exact Differential Equation

(12) Solve: $\sin px \cos y = \cos px \sin y + p$

Q.3

(a) State and prove Leibnitz's theorem. [05]

(b) For $y = \log(ax+b)$, prove that $y_n = \frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ [05]

OR

Q.3

(a) In usual notations prove that, $\tan\theta = \dfrac{r}{\dfrac{dr}{d\theta}}$. [05]

(b) If $x = \cos(\frac{1}{m}\log y)$ then find $y_n(0)$. [05]

Q.4

(a) Find the length of arc of the parabola $y^2=4ax$ (a>0) measured from the vertex to one extremity of its latus rectum. [05]

(b) Find the intrinsic equation of the cardioid $r=a(1+\cos\theta)$. Hence prove that $s^2+9\rho^2=16a^2$, where ρ is the radius of curvature at any point of the curve. [05]

OR

Q.4

(a) Show that the radius of curvature at any point of the curve $x = ae^{\theta}(\cos\theta - \sin\theta), y = ae^{\theta}(\sin\theta + \cos\theta)$ is twice the perpendicular distance of the tangent at the point from the origin. [05]

(b) Show that the intrinsic equation of the curve $y^3=ax^2$ is $27s=8a(\sec^3\psi-1)$. [05]

Q.5

(a) State and prove Euler's theorem for homogeneous function of three variables. [05]

(b) If $z = f(x,y), x = r\cos\theta, y = \sin\theta,$ then [05]

prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

OR

Q.5

(a) Verify Euler's theorem for $z = x^n \log\left(\frac{y}{x}\right)$ [05]

and find $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}$

(b) If $z = xy\, f(\frac{y}{x})$ and z is constant, then show that $\dfrac{f'\left(\frac{y}{x}\right)}{f\left(\frac{y}{x}\right)} = \dfrac{x\left[y + x\frac{dy}{dx}\right]}{y\left[y - x\frac{dy}{dx}\right]}$ [05]

Q.6 Prove that the necessary and sufficient condition for the differential equation Mdx+Ndy = 0 to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [10]

OR

Q.6 Solve: $(p+y+x)(xp+x+y)(p+2x)=0$ [10]

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