SARDAR PATEL UNIVERSITY
B.Sc.(I SEM.) (NC) EXAMINATION
Thursday, 27th December, 2012
10.30 am to 12.30 pm
US01CMTH02 : MATHEMATICS
Calculus and Differential Equation

Total Marks: 70

Note: Figures to the right indicate full marks of the questions.

Q.1 Answer the following questions by selecting the most appropriate option. Write down the option in your answer book. [10]

(1) If $y = e^{5x}$ then y_n = ___________
(a) e^{5x} (b) $5e^{5x}$ (c) $5^n e^{5x}$ (d) $5^n e^x$

(2) If y = sin7x then y_n = ___________
(a) $7^n \sin(7x + \frac{\pi}{2})$ (b) $7^n \sin(7x + \frac{n\pi}{2})$
(c) $7^n \cos(7x + \frac{\pi}{2})$ (d) $7^n \cos(7x + \frac{n\pi}{2})$

(3) For a polar curve $r=f(\theta)$, $\tan\phi$ = __________
(a) $\frac{r_1}{r_2}$ (b) $\frac{r_2}{r_1}$ (c) $\frac{r_1}{r}$ (d) $\frac{r}{r_1}$

(4) $\sqrt{r^2 + (\frac{dr}{d\theta})^2}$ = ___________
(a) $\frac{ds}{d\theta}$ (b) $\frac{ds}{dr}$ (c) ϱ (d) $\frac{1}{\varrho}$

(5) For a polar curve, ϱ = _____________
(a) $\frac{(r^2+r_1^2)^{\frac{3}{2}}}{r^2+2r_1^2-rr_2}$ (b) $\frac{(r^2+r_2^2)^{\frac{3}{2}}}{r^2+2r_1^2-rr_2}$
(c) $\frac{(1+r_1^2)^{\frac{3}{2}}}{r_2}$ (d) $\frac{(1+r_2^2)^{\frac{3}{2}}}{r_1}$

(6) $\frac{\partial^2 z}{\partial y \partial x}$ = ___________
(a) $\frac{\partial z}{\partial y} \cdot \frac{\partial z}{\partial x}$ (b) $\frac{\partial^2 z}{\partial y^2}$ (c) $\frac{\partial}{\partial y}(\frac{\partial z}{\partial x})$ (d) $\frac{\partial^2 z}{\partial x^2}$

(7) Let z = f (x,y) be defined on E, then $\frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$ = __________
(a) $\frac{dz}{dx}$ (b) dz (c) $\frac{dz}{dy}$ (d) $\frac{dy}{dx}$

(8) $u = \frac{x+y}{x-y}$ is homogeneous function of degree __________
(a) 3 (b) 1 (c) 2 (d) 0

(9) The differential equation $(x^2 + 2xy - y^2)dx - (x+y)^2 dy = 0$ is ___________
(a) Exact (b) Not Exact (c) Clairaut's (d) None of these

(10) The general solution of the differential equation y=px+7p is _____________
(a) y=x+7 (b) y=cx+7c (c) cx+7c=0 (d) y=cp+7c

Q.2 Write down answers of ANY TEN questions in short. [20]

(1) Find nth derivative of $y = e^{mx}$
(2) If y = cos (ax+b) then find y_n.
(3) For $y = a^{mx}$, find y_n.
(4) Find the radius of curvature of any point on the curve $r_1=a\theta$.

(5) Determine the values of $\boldsymbol{\theta}$ at points of intersection of r = a(1+cos$\boldsymbol{\theta}$) and r = -a cos$\boldsymbol{\theta}$.

(6) Find $\frac{ds}{dx}$ for y = a cosh$\left(\frac{x}{a}\right)$

(7) Define: Homogeneous Function

(8) State Euler's theorem for function of three variables.

(9) Define: Implicit Function

(10) Define: Orthogonal Trajectory

(11) Solve: sinpx cosy = cosp xsiny + p

(12) Define: Exact Differential Equation

Q.3

(a) If y =y e^{ax} sin (bx+c) then find y_n. [05]

(b) Find angle between radius vector and tangent to the curve $r^m=a^m$ (cosm $\boldsymbol{\theta}$+sinm $\boldsymbol{\theta}$) [05]

OR

Q.3

(a) If y =$(\boldsymbol{x}-\sqrt{\boldsymbol{4}+\boldsymbol{x}^2})^m$, then find $y_n(0)$. [05]

(b) Obtain the angle between radius vector and tangent for the polar curve r = f($\boldsymbol{\theta}$). [05]

Q.4

(a) In usual notations prove that, $\varrho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ [06]

(b) Prove that the length of the curve $\boldsymbol{x}^{\frac{2}{3}}+\boldsymbol{y}^{\frac{2}{3}}=\boldsymbol{a}^{\frac{2}{3}}$ meausured from (0,a) to the point (x,y) is $\frac{3}{2}\ (\boldsymbol{a}\boldsymbol{x}^2)^{\frac{1}{3}}$ [05]

OR

Q.4

(a) For a cartesian curve y = f(x), prove that $\frac{ds}{dx}=\sqrt{\mathbf{1}+\left(\frac{dy}{dx}\right)^2}$ [05]

(b) Show that the intrinsic equation of the curve $y^3 = ax^2$ is 27s = 8a($\sec^3\boldsymbol{\psi}-\mathbf{1}$)

Q.5

(a) State and prove Euler's theorem for homogeneous function of two variables. [05]

(b) If H = f(2x-3y, 3y-4z, 4z-2x), then prove that $\frac{1}{2}\frac{\partial H}{\partial x}+\frac{1}{3}\frac{\partial H}{\partial y}+\frac{1}{4}\frac{\partial H}{\partial z}=\mathbf{0}$ [05]

OR

Q.5

(a) Verify Euler's theorem for z = $\boldsymbol{x^n log}\left(\frac{y}{x}\right)$ [05]

(b) If A, B, C ar angles of a ΔABC such that $\sin^2$A+$\sin^2$B+$\sin^2$c = K, a constant then prove that $\frac{dB}{dC}=\frac{tnaC-tanA}{tanA-tanB}$ [05]

Q.6 Prove that the necessary and sufficient condition for the differential equation Mdx+Ndy = 0 to be exact is that $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ [10]

OR

Q.6 Solve: (p+y+x) (xp+x+y) (p+2x) = 0 [10]

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