Se

[74]

No. of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc. (I Semester) Examination
12th April 2016 (Tuesday)
2.30 pm – 4.30 pm
MATHEMATICS
US01CMTH02 – Calculus and Differential equations

Total Marks : 70

Q.1 Answer the following by selecting the correct answer from the given options. (10)

(1) If y= log(ax+b) then y_n=______

(a) $\frac{(-1)^n n!\, a^n}{(ax+b)^{n+1}}$ (b) $\frac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n}$ (c) $\frac{(-1)^n n!\, a^n}{(ax+b)^n}$ (d) $\frac{(-1)^{n-1} n!\, a^n}{(ax+b)^n}$

(2) If y= cos3x then y_n=______

(a) $3^n\cos(3x+\frac{\pi}{2})$ (b) $3^n\cos(3x+\frac{n\pi}{2})$ (c) $3^n\sin(3x+\frac{n\pi}{2})$ (d) $3^n\sin(3x+\frac{\pi}{2})$

(3) If $y=(x^2-2)^m$ then $(x^2-2)y_1$ = ______

(a) mxy (b) 2my (c) 2mx (d) 2mxy

(4) For r=f(θ) which of the following is not true ?

(a) $\frac{ds}{d\theta}=\sqrt{1+(\frac{dr}{d\theta})^2}$ (b) $tan\emptyset=\frac{r}{r_1}$

(c) $\rho=\frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2+2{r_1}^2-rr_2}$ (d) $\frac{ds}{d\theta}=\sqrt{r^2+(\frac{dr}{d\theta})^2}$

(5) let x=x(t) and y=y(t) be the parametric equation of the curve then $\frac{ds}{dt}$= ______

(a) $\sqrt{1+(\frac{dx}{dt})^2}$ (b) $\sqrt{1+(\frac{dy}{dt})^2}$ (c) $\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}$ (d) $\sqrt{1+(\frac{dy}{dx})^2}$

(6) Curvature of the line $2x+3y=1$ is______

(a) 1 (b) 0 (c) $\frac{-2}{3}$ (d) $\frac{3}{2}$

(7) f(x,y) = $x^2y^4-x^3y^3+xy^5$ is a homogeneous function of degree ______

(a) 6 (b) 4 (c) 0 (d) 5

(8) If f(x, y) = $5x^4-4xy^3$ then $\frac{\partial^2 f}{\partial x\partial y}$ = ______

(a) $-12y^2$ (b) $60x^2-12y^3$ (c) $20x^3-12xy^2$ (d) $20x^3-4y^3$

(9) Order and degree of $x^2{y_2}^3+y{y_1}^5+xy=0$ are ________respectively

(a) 3 & 2 (b) 1 & 5 (c) 2 & 3 (d) 3 & 5

(10) To solve Clairaut's equation replace p by______

(a) p (b) $-p$ (c) $constant$ (d) 0

(P.T.O)

Q.2 **Answer ANY TEN of the following.** (20)

(1) If y=$sin(ax+b)$ then find y_n.

(2) If y=$xsinx$ then find y_n.

(3) Find $\emptyset$ for the curve r=a(1+θ).

(4) Find $\frac{ds}{dx}$ for y $= a\ sin2x$.

(5) Define: (1) Average curvature (2) intrinsic equation.

(6) Find the length of the curve $y = 3x - 9$ measured from (0,-9) to (3,0) .

(7) Let u=u(x, y) be a non-homogeneous function and z=$\emptyset$(u) homogenous then Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{\emptyset(u)}{\emptyset'(u)}$.

(8) For $u = x^3 - 3xy^2$ find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

(9) Verify Euler's theorem for $z = x^2y - xy^2$

(10) Find the solution of the differential equation $px = y - p^2$.

(11) Find the orthogonal trajectories of family of curve $y = cx$ where c is the Parameter.

(12) Determine whether $x^3y\ dx - xy^3\ dy = 0$ is exact or not.

Q.3

(a) State and prove Leibniz's theorem. (05)

(b) For $y = e^{ax}\sin(bx + c)$, then prove that $y_n = r^n e^{ax}\sin(bx + c + n\emptyset)$ (05)

Where $r = \sqrt{a^2 + b^2}$, $\emptyset = tan^{-1}(\frac{b}{a})$

OR

Q.3

(a) Find y_n for $y = e^{2x}cosxsin^2 2x.$ (05)

(b) If $x = cos(\frac{1}{m}logy)$ then find $y_n(0)$. (05)

Q.4

(a) Fix a point A(x_0,y_0) on a curve given by y=f(x). For a point P(x,f(x)) on the curve ,let s be the arc length of arc AP. Then prove that $\frac{ds}{dx} = \sqrt{1 + (\frac{dy}{dx})^2}$ (05)

(b) Show that the entire length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$. (05)

OR

Q.4

(a) Let y=f(x) be a curve and P be a point then prove that the radius of curvature at P is given by $\rho = \frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$ where $y_1 = \frac{dy}{dx}$ & $y_2 = \frac{d^2y}{dx^2}$ (05)

(b) Find the intrinsic equation of the Cardioid $= a(1 + cos\theta)$. Hence prove that $s^2 + 9\rho^2 = 16a^2$. Where ρ is radius of curvature at any point of the curve. (05)

Q.5

(a) State and prove Euler's theorem for homogeneous function of two variables. (05)

(b) If $z = xyf(\frac{y}{x})$ and z is constant then prove that $\frac{f'(\frac{y}{x})}{f(\frac{y}{x})} = \frac{x[y+x\frac{dy}{dx}]}{y[y-x\frac{dy}{dx}]}$. (05)

OR

Q.5

(a) If $z = f(x,y)$, $x = rcos\theta$, $y = rsin\theta$,then prove that (05)

$$(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2$$

(b) If A,B and C are angles of a ΔABC such that $sin^2A + sin^2B + sin^2C = K$,a constant, then prove that $\frac{dB}{dC} = \frac{\tan C - \tan B}{\tan A - tanB}$. (05)

Q.6 Prove that the necessary and sufficient condition for the differential equation Mdx+Ndy=0 to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. (10)

OR

Q.6 Solve $(p + y + x)(xp + x + y)(p + 2x) = 0$. (10)

—— X ——