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No. of printed pages: 3

**SARDAR PATEL UNIVERSITY**  
**B.Sc. (I Semester) Examination**  
**2016**

11<sup>th</sup> April 2016 (Monday)

2.30 pm - 4.30 pm

**US01CMTH01 – Mathematics / Analytic Geometry and Complex Numbers**

**Total Marks: 70**

**Q.1 Answer the following by selecting the correct answer from the given (10) options.**

- 1) Asymptote of  $y = x^3 - 3x^2 + 2x$  are \_\_\_\_\_.  
 (a)  $x = 0, 1, 2; y = 1$  (b)  $x = 0, -1, 2; y = 0$  (c)  $x = 0, 1, -2$  (d) Not possible
- 2) If  $(\frac{dy}{dx})_p = 0$  then the tangent at point p is parallel to the \_\_\_\_\_.  
 (a) y-axis (b) x-axis (c) Line  $x = 5$  (d) None of these
- 3) The shape of Lemniscates looks like \_\_\_\_\_.  
 (a) 8 (b) Flower (c) Rose (d) Heart Shape
- 4) The curve of  $r = 5 \sin 7\theta$  has \_\_\_\_\_ loops.  
 (a) 5 (b) 7 (c) 14 (d) 12
- 5) The curve  $r = a\theta$  is symmetric about \_\_\_\_\_.  
 (a) Polar axis (b) Normal axis (c) pole (d) None of these
- 6) Polar equation of vertical line through the point  $(3, 180^\circ)$  is \_\_\_\_\_.  
 (a)  $3 = r \cos \theta$  (b)  $3 = r \sin \theta$  (c)  $3 = -r \sin \theta$  (d)  $3 = -r \cos \theta$
- 7) The equation of curve  $3 = r(\cos \theta - 2 \sin \theta)$  represent \_\_\_\_\_.  
 (a) Circle (b) Line (c) Hyperbola (d) Parabola
- 8) If eccentricity  $e > 1$  then conic is \_\_\_\_\_.  
 (a) Hyperbola (b) Parabola (c) Circle (d) ellipse
- 9) The imaginary part of  $z = (1+2i)(1-3i)$  is \_\_\_\_\_.  
 (a) 1 (b) -1 (c) 7 (d) -7
- 10)  $(\text{cis}\theta)^{3/7}$  has only \_\_\_\_\_ distinct values.  
 (a) 3 (b) 7 (c)  $\frac{3}{7}$  (d)  $\frac{1}{7}$

**Q.2 Answer ANY TEN of the following.**

**(20)**

- 1) Discuss symmetries of the curve  $xy - 16 = 0$ .
- 2) Find the parametric equation of  $x^{2/3} + y^{2/3} = a^{2/3}$ .
- 3) Find the tangent parallel to axes for  $x = \cos^2\theta; y = 2\sin\theta$ .
- 4) Identify the curve  $3(1 - \cos\theta)$ .
- 5) Transfer the equation  $r = \tan\theta + \sec\theta$  in to Cartesian form.
- 6) Express the point  $(\sqrt{3}, 1)$  in polar form.

- 7) Find radius and center of the circle  $r = 3\sin \theta$ .
- 8) Find the equation of the horizontal line through the point  $(-2, 90^\circ)$ .
- 9) Find the polar equation of circle with center at  $(4, 90^\circ)$  and passes through pole.
- 10) Find modulus of  $z = \frac{(3-\sqrt{2}i)^2}{1+2i}$ .
- 11) Find amplitude of  $z = -\sqrt{3} + i$ .
- 12) Find  $z + \frac{1}{z}$  for  $z = 4+5i$ .

**Q.3**

- a) If a curve given by  $x = f(t)$ ,  $y = g(t)$  and both  $x$  and  $y$  get numerically large as  $t$  approaches some number say "a". Then an oblique asymptote to the curve if it exist is given by  $y = mx + c$ ; (05)  
 Where,  $m = \lim_{t \rightarrow a} \left(\frac{dy}{dx}\right)$  and  $c = \lim_{t \rightarrow a} (y - mx)$ .
- b) Sketch the curve given by  $y = \frac{x(x-4)}{(x-1)(x+2)}$  (05)

**OR**

**Q.3**

- a) Obtain parametric equations of cycloid. (05)
- b) Find the equations of tangent and normal line to a curve  $y^2 = 4ax$  at point  $(at^2, 2at)$ . (05)

**Q.4**

- a) State when the polar curve is symmetric with respect to polar axis? Also prove it. (05)
- b) Discuss symmetry, extent, closeness of the curve  $r^2 = 9\sin 2\theta$  and hence sketch the curve. (05)

**OR**

**Q.4**

- a) State when the polar curve is symmetric with respect to normal axis? Also prove it. (05)
- b) Sketch the curve given by  $r = 3(1 + \cos \theta)$  (05)

Q.5

- a) In usual notations prove that  $r = \frac{pe}{1+e \cos \theta}$ . (05)
- b) If any straight line through the pole meets the circle  $r^2 - 2rd \cos(\theta - \alpha) + d^2 - a^2 = 0$  at point P and Q. (05)  
Then prove that  $OP \cdot OQ = d^2 - a^2$ .

OR

Q.5

- a) Prove that equation of line not passing through the pole is  $p = r \cos(\theta - \omega)$ , where  $(p, \omega)$  is the foot of perpendicular from the pole. (05)
- b) Identify curve  $r = 1 + \cos \theta$ . Also find its reciprocal curve. Sketch both of the curves with the same frame of reference. (05)

Q.6

State and prove De-Moiver's Theorem. (10)

OR

Q.6

Expand  $\cos^8 \theta$  in a series of cosines of multiples of  $\theta$ . (10)

X=X=X