[67/A15]

SEAT No.________ **No. of Printed Pages : 3**

SARDAR PATEL UNIVERSITY

BSc Examination [Semester: V]

Subject: Physics Course: US05CPHY02

Mathematical Physics

Wednesday, Date 24-10-2018 Time: 10.00 am to 1.00 pm Total Marks: 70

INSTRUCTIONS:

1 Attempt all questions.
2 The symbols have their usual meaning.
3 Figures to the right indicate full marks.

Q-1 Multiple Choice Questions: [Attempt all] **10**

(i) The orthogonality condition for curvilinear co-ordinates is ______.

(a) $\frac{\partial r}{\partial u} \cdot \frac{\partial r}{\partial u} = 0$ (b) $\frac{\partial r}{\partial u} \cdot \frac{\partial u}{\partial v} = 0$
(c) $\frac{\partial u}{\partial r} \cdot \frac{\partial v}{\partial r} = 0$ (d) $\frac{\partial r}{\partial u} \cdot \frac{\partial r}{\partial v} = 0$

(ii) A square matrix A is called singular matrix if $|A|$ =_____.

(a) 0 (b) 1
(c) ∞ (d) -1

(iii) For a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 4 & 6 \end{bmatrix}$, $|A|$ =_____.

(a) 2 (b) 1
(c) 0 (d) -1

(iv) For Bessel's equation, _______.

(a) $k = 1$ or $k = -1$ (b) $k = n$ or $k = -n$
(c) $k = n$ or $k = n - 1$ (d) $k = n$ or $k = -n - 1$

(v) The generating function for Hermite polynomial is _______.

(a) e^{x} (b) e^{2tx-t^2}
(c) $e^{\frac{x}{2}(t-1)}$ (d) e^{2x-t^2}

(vi) $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2vy = 0$ is called _________. (Here v is a parameter)

(a) Hermite differential equation (b) Legendre's differential equation
(c) Bessel's differential equation (d) First order differential equation

(vii) The coefficients a_0 for Fourier series of a periodic function $f(x)$ in $[-\pi, \pi]$ =_____.

(a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$ (b) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx$

(c) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \, dx$ (d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \, dx$

(viii) The amount of heat ΔH crossing an element of surface ΔS in time Δt is given by

(a) $\Delta H = K\,\Delta S\,\left|\frac{du}{dt}\right|$ (b) $\Delta H = K\,\Delta S\,\Delta t\,\left|\frac{du}{dt}\right|$

(c) $\Delta H = K\,\Delta t\,\left|\frac{du}{dt}\right|$ (d) $\Delta H = K\,\frac{\Delta S}{\Delta t}\left|\frac{du}{dt}\right|$

(ix) "The best representative curve to the given set of observations is one for which E, the sum of the squares of the residuals, is minimum". This concept is known as ______.

(a) Principle of least squares (b) Principle of differentiation

(c) Principle of integration (d) None of these

(x) The backward difference operator ∇ defined as

(a) $\nabla y_i = y_{i-1} - y_i$ (b) $\nabla y_i = y_i - y_{i-1}$

(c) $\nabla y_i = y_{i+1} - y_i$ (d) $\nabla y_i = y_i - y_{i+1}$

Q-2 Answer the following questions in short. (Attempt any ten) **20**

(1) Define (i) symmetric matrix and (ii) inverse of a matrix

(2) Determine eigen values for matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$. (Use relation: $|A - \lambda I| = 0$)

(3) Define curvilinear co-ordinates.

(4) Write Bessel's differential equation.

(5) For Bessel's function, prove that: $x\,J_n'(x) = n\,J_n(x) - x\,J_{n+1}(x)$.

(6) Show that $P_n(-\mu) = (-1)^n\,P_n(\mu)$.

(7) Define Fourier series.

(8) Write the Fourier equation of heat flow.

(9) Write cosine series for $f(x)$ when $0 \le x \le \pi$. (Note: derivation is not required)

(10) Define (i) interpolation and (ii) extrapolation.

(11) For a shift operator E, show that $\Delta = E - 1$.

(12) Derive an equivalent equation of a straight line for $y = ae^{bx}$.

Q-3 (a) Derive expression of gradient in terms of orthogonal curvilinear system. **6**

(b) If $u = 3x + 2, v = y - 1, w = 5z + 3$, show that u, v and w are orthogonal and find ds^2 and the metrical coefficients: h_1, h_2, h_3. **4**

OR

Q-3 (a) Discuss spherical polar co-ordinates as a special curvilinear system. **6**

(b) Discuss orthogonal curvilinear coordinates and show that $\left[\frac{\partial \vec{r}}{\partial u}\,\frac{\partial \vec{r}}{\partial v}\,\frac{\partial \vec{r}}{\partial w}\right] = h_1 h_2 h_3$ **4**

Q-4 (a) State and derive the Rodrigue's formula. 6

(b) Show that for Bessle's function: 4

$$e^{\left(\frac{x}{2}\right)\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$$

OR

Q-4 (a) Derive the series solution of Legendre differential equation in the form of descending power of x. 6

(b) For Hermite polynomial using equation $H_n(x) = e^{x^2}(-1)^n \frac{d^n}{dx^n}\left(e^{-x^2}\right)$, calculate Hermite polynomials $H_0(x),\ H_1(x)$. 4

Q-5 (a) Derive the Fourier series for a complex periodic function $f(t)$ defined in $(-\infty, \infty)$. Also find the coefficients α_n and β_n. 6

(b) Write a note on application of Fourier series involving phase angles. 4

OR

Q-5 (a) Obtain Fourier series for $f(x) = x \cdot \sin x$ in the interval $-\pi < x < \pi$. show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots$ 6

(b) Derive one dimensional diffusion equation for one dimensional flow of electricity in a long insulated cable. 4

Q-6 Derive Newton's forward difference interpolation formula and evaluate $f(12)$ using the values given in the following table of values. 10

x	10	20	30	40	50
$y = f(x)$	46	66	81	93	101

OR

Q-6 Using trapezoidal rule and Simpson's 1/3 rule compute the approximate value of $I = \int_0^{\pi} \sin x \, dx$ by dividing the range of integration into ten equal parts so that each part is of width $\frac{\pi}{10}$. What is the analytical value of $I = \int_0^{\pi} \sin x \, dx$. Which method gives more accurate result? 10

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