[39/A-8]

No of printed pages: 3

SARDAR PATEL UNIVERSITY B.Sc. (SEMESTER - V) EXAMINATION Thursday, 1st November, 2018 MATHEMATICS: US05CMTH05

(Number Theory)

$\Gamma ime: 10:00 \ a.m. \ to \ 01:00 \ p.m$	Γ ime	:	10:00	a.m.	to	01:00	p.m
---	--------------	---	-------	------	----	-------	-----

Maximum Marks: 70

Prove that the successive l'Illianora nucliers ar

(a) If ca = ib(mod n) and (can) = 1 then pr(b)

Que.1 Fill in the blanks.

10

 $(1) (a, 1) = \dots, \forall a \in \mathbb{Z}$

$$(1) (a, 1) = \dots, \forall a \in \mathbb{Z}$$

(a)
$$-a$$
 (b) $|a|$ (c) a (d)

(2)
$$(a, b)[a, b] = \dots \forall a, b \in \mathbb{Z}$$
.

$$(3) [25, 30] = \dots$$

(4) $P(60) = \dots$

(a)
$$120$$
 (b) 60 (c) 60^6 (d) 60^5

(5) is a Mersenne number of the prove that we will be used a Mersenne number of the state of the state

(6)
$$\mu$$
(6) =

(a)
$$1 = (b) = 0 + (c) + 1 + (d) = 2$$

(7) Prove that every number containing more than three digits can be divided by 8 iff the number formed by digits can be divided by 8. To memoral lamentaling around him and (b)

Que 3 (c) Let g be a positive innegar greater than prove that every positive integer a can

- (a) last two (b) last three (d) first three first two
- (8) 765432 is not divisible by

(9) If $a^d \equiv 1 \pmod{m}$ and n is order of a modulo m then

(a)
$$n < d$$
 (b) d/n (c) $d = n$ (d) n/d

(10) $2x + 4y \equiv 5 \pmod{12}$ has only solutions.

(a) 1 (b)
$$24$$
 (c) 12 (d) 6 5 miles formed that every result is 6 (d, a) If (a) 6 eV.

Que.2 Answer the following (Any Ten)

20

- (1) Prove that if (a, b) = d then $\exists x, y \in \mathbb{Z}$ such that xa + yb = d.
- (2) Prove that (a, b) = (b, a + kb), for $k \in \mathbb{Z}$.
- (3) Prove that common multiple of a and b is a multiple of their lcm.
- (4) Let x be any positive real number and n be any positive integer then prove that among the integers from 1 to x the number of multipliers of n is $\left[\frac{x}{n}\right]$.
- (5) Prove that the successive Fibonacci numbers are relatively prime.
- (6) Prove that $u_{n+1}^2 = u_n^2 + 3u_{n-1}^2 + 2[u_{n-2}^2 + u_{n-3}^2 + \dots + u_1]$.
- (7) If $a_1 \equiv b_1 \pmod{n}$ then prove that $a_1^m \equiv b_1^m \pmod{n}$, $\forall m \in \mathbb{N}$.
- (8) If $ca \equiv cb \pmod{n}$ and (c,n) = 1 then prove that $a \equiv b \pmod{n}$.
- (9) Prove that Pythagoras equation has no prime solution .
- (10) Solve the equation $12x + 15 \equiv 0 \pmod{45}$.
- (11) Find $\phi(243) + \phi(81) + \phi(27) + \phi(9) + \phi(3)$.
- (12) If (a,p)=1, p is prime, then prove that $a^{p-1}\equiv 1 \pmod{p}$.
- Que.3 (a) If P_n is n^{th} prime number then prove that $P_n < 2^{2^n}$, $\forall n \in \mathbb{N}$.
 - (b) State and prove unique factorization theorem for positive integers.

OR

- Que.3 (c) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a=c_ng^n+c_{n-1}g^{n-1}+\ldots\ldots+c_1g+c_0$, where $n\geq 0$, $c_i\in\mathbb{Z}$, $0\leq c_i< g$, $c_n\neq 0$.
 - (d) State and prove Fundamental theorem of divisibility . It and an action and bondered a
- Que.4 (a) In usual notation prove that $P(n!) = \sum_{k=1}^{m} \left[\frac{n}{p^k} \right]$, where $p^m \le n < p^{m+1}$. Hence find 2(50!), 4(50!) and 8(50!).
 - (b) Prove that g.c.d of two Fibonacci numbers is also a Fibonacci number.

${}^{t}OR^{t} = 1(mod \, m)$ and n is order of a model of (0, 1)

- Que.4 (c) Prove that every prime factor of F_n (n > 2) is of the form $2^{n+2}t + 1$, for some integer t.
 - (d) Prove that $S(a) < a\sqrt{a}$, $\forall \ a > 2$. The second of the second
- Que.5 (a) If (a,b)=d, then prove that general solution of ax+by=c can be written as $x=x_0+\frac{b}{d}$ t; $y=y_0-\frac{a}{d}$ t, where $t\in\mathbb{Z}$ and $x=x_0$, $y=y_0$ is a particular solution of ax+by=c.
 - (b) Find general solution of equation 50x + 45y + 36z = 10.

(c) Find positive integer solution of equation $y - \frac{x+3y}{x+2} = 1$.

Que.5 (d) Prove that $x^4 + y^4 = z^2$ has no nonzero positive integer solution. Hence prove that $x^{-4} + y^{-4} = z^{-4}$ has no nonzero positive integer solution.

(e) Prove that a general integer solution of $x^2+y^2+z^2=w^2$, (x,y,z,w)=1 is given by $x=(a^2-b^2+c^2-d^2)$, y=2ab-2cd, z=2ad+2bc, $w=a^2+b^2+c^2+d^2$.

5

Que.6 (a) State and prove Sun-Tsu theorem . Hence solve the system $x\equiv -2 (mod~12)~~;~~x\equiv 6 (mod~10)~~;~~x\equiv 1 (mod~15)$.

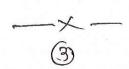
(b) In usual notation prove that $\sum_{i=0}^{k} \Phi(p^{i}) = p^{k}$, where p is prime .

OR

- Que.6 (c) Prove that $ax + b \equiv 0 \pmod{m}$, where (a, m) = d, d > 1 has solution iff d/b.

 Also prove that it has d solutions $x_i \equiv a + i \frac{m}{d} \pmod{m}$, $i = 0, 1, 2, \dots, d-1$,

 of which $x \equiv a \pmod{\frac{m}{d}}$ is unique solution of $\frac{a}{d}x + \frac{b}{d} \equiv 0 \pmod{\frac{m}{d}}$.
 - (d) Prove that Euler's function is multiplicative function .



- (c) Find positive into an adultion of equation $y = \frac{x+1/y}{x+2} = 1$
- Upon 1 (d) Prove that $x^3 + y^4 = z^2$ has no nonzero posity a rate of stion. Hence prove that $e^{-x} + y^3 = z^{-2}$ has no maxera positive integers, artison.
 - (c) From that a second integer solution of $x^2+y^2+z^2=w^2$, (e.g. x,w)=1 is given by $x=(a^2-b^2+c^2-b^2)$, y=2ab-3d, z=2ad+2b, $w=c^{-1}b^2+c^2+d^2$.
 - Que.C. (a) State and prove San Tan theorem . Hence solve the symmatric = -2(m+12) : $x \equiv 0 \pmod{10}$
 - (ii) In axial notation prove that $\sum \Phi(\rho) = \rho^{k}$, where ρ is prime.

110

One if (i) Prive that $ax+b\equiv 0$ (is or in), where ax+d=d>1 has solution iff axb

of which
$$x \equiv a \left(mod \frac{m}{d} \right)$$
 is unitary solution of $\frac{a}{a} = 0 \pmod{\frac{m}{a}}$.

(d) Prove that Euler's fair that is a which which the