8c

SEAT No.________

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[59/A-14]

SARDAR PATEL UNIVERSITY

B.Sc.SEM-V EXAMINATION

29^{th} October 2018 , Monday

10.00 a.m. to 01.00 p.m.

US05CMTH04 (Abstract Algebra-I) Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Every infinite cyclic group has exactly .... generators .
(a) 3 (b) 1 (c) 2 (d) 4

(2) Centre of $\mathbb{Z}$ is ....
(a) $\mathbb{Z}$ (b) 2 (c) N (d) 1

(3) In Klein 4-group $G = \{e, a, b, c\}$, $b^2 = \ldots$
(a) e (b) b (c) c (d) a

(4) A permutation σ is said to be odd permutation if signature of σ is ....
(a) 2 (b) -1 (c) 1 (d) -2

(5) ——————— is generator of group Z_5^*
(a) $\bar{3}$ (b) $\bar{1}$ (c) $\bar{4}$ (d) $\bar{5}$

(6) External direct sum of Z_2 is ....
(a) Klein 4- group (b) Q (c) Z (d) Z_2

(7) Multiplicative inverse of 5 in Z_7^* is ....
(a) 3 (b) 6 (c) 2 (d) 1

(8) Every noncyclic group of order 4 is isomorphic to ——
(a) Klein 4-group (b) Z (c) N (d) Z_4

(9) $O(-i)$ in $\{\pm 1, \pm i\}$ is ....
(a) 3 (b) 4 (c) 1 (d) 2

(10) Every group of order ——— is abelian group .
(a) 2 (b) 5 (c) 4 (d) 6

Q.2 Attempt the short questions. [any ten] [20]

(1) Prove that every subgroup of Z is of the form nZ,for some $n \in Z$.
(2) Prove that $(M_2(R), +)$ is a group.
(3) Prove that intersection of two subgroups of a group G is also a subgroup of G.
(4) Find $O(2)$ in Z.
(5) State and prove left cancelation law in group G.
(6) Let H be any subgroup of group G. Then prove that $aH = H \Leftrightarrow a \in H$.
(7) Prove that $\theta : Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z.
(8) Express the inverse of cycle (12453) as a product of transpositions.
(9) Define Group and finite group.
(10) Is the product of permutation of 4 symbols commutative? Justify.
(11) Prove that S_n is a finite group of order $n!$.
(12) Prove that $Z(G)$,the centre of group G is a subgroup of G. (PTO)

Q.3(a) Let H and K be subgroups of group G.Then prove that HK is subgroup of G iff $HK = KH$. [6]

(b) Check whether the set $(Z, *)$, where $*$ is defined as $a*b = a+b-ab \ \forall \ a, b \in Z$ forms a group or not. Verify it. Is it commutative ? . [4]

OR

Q.3(c) Let H and K be finite subgroups of group G such that HK is a subgroup of G. Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. [6]

(d) Prove that intersection of any number of subgroups of a group G is also a subgroup of G. [4]

Q.4(a) Let G be a cyclic group and H is a subgroup of G. Show that H is also cyclic. [5]

(b) Let G be a finite cyclic group of order n ,then prove that G has $\phi(n)$ generators. [5]

OR

Q.4(c) Prove that every subgroup of cyclic group is also cyclic. [5]

(d) If G is cyclic group of order n and $a^m = e$ for some $m \in \mathbb{Z}$ then prove that n/m. [5]

Q.5(a) Let $G = (a)$ be a finite cyclic group of order n. Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n. [6]

(b) Prove that a homomorphism $\theta : G \to G'$ of G to G' is an one-one iff $Ker\theta = \{e\}$. [4]

OR

Q.5(c) State and Prove First isomorphism theorem. [6]

(d) Prove that every infinite cyclic group has only one non-trivial automorphism. [4]

Q.6(a) Prove that S_n is non commutative group of order $n!$. [6]

(b) If G is a direct product (internal) of subgroups H and K , then Prove that G is isomorphic to the external direct product of H and K. [4]

OR

Q.6(c) State and prove Cayley's theorem. [6]

(d) Prove that the set S_n of all permutation on n symbols forms a non-commutative group. [4]

—x—