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(66 & A-13) SEAT No.________

No. of Printed Pages : 3

Sardar Patel University,

B.Sc. Examinations: 2018-19 – V SEM

Subject : Mathematics US05CMTH02 Max. Marks : 70

Real Analysis-II

Date: 24/10/2018, Wednesday Timing: 10.00 am - 01.00 pm

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] If a sequence $\{S_n\}$ is such that $\lim_{n \to \infty} \frac{S_{n+1}}{S_n} = l$ then $\lim_{n \to \infty} S_n = \infty$ if
[A] $l < 1$ [B] $l \leqslant 1$ [C] $l > 1$ [D] $l \geqslant 1$

[2] The sequene $\{2^n\}$
[A] is convergent [B] diverges to ∞ [C] diverges to $-\infty$ [D] oscillates finitely

[3] A sequence $\{S_n\}$; where

$$S_n = \begin{cases} 2 & ; \text{ if } n = 1 \text{ or even} \\ p & ; \text{ where } p \text{ is the smallest prime factor of n.} \end{cases}$$

is
[A] convergent [B] divergent [C] oscillates finitely [D] oscillates infinitely

[4] A positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if
[A] $p < 1$ [B] $p > 1$ [C] $p \leqslant 1$ [D] $p \geqslant 1$

[5] The positive term series $1 + r + r^2 + r^3 + ... + r^n + ...$ converges for
[A] $r > 1$ [B] $r \geqslant 1$ [C] $r < 1$ [D] $r \leqslant 1$

[6] If $\lim_{n \to \infty} u_n = 1$, for a positive term series $\sum_{n=1}^{\infty} u_n$ then the series
[A] converges to 0 [B] converges to 1 [C] converges to 2 [D] cannot converge

[7] For $f(x, y) = x^3 - 3xe^y$ the value of $f_x(1, 0)$ is
[A] 0 [B] 1 [C] 2 [D] 3

[8] $\lim_{(x,y) \to (0,0)} \frac{x \sin(x^2 + y^2)}{x^2 + y^2} =$
[A] 0 [B] 1 [C] 2 [D] 3

[9] For a function f, if $f_x(a, b) < f_y(a, b)$ then at (a, b), f has
[A] no extreme value [B] a minimum [C] a maximum [D] a stationery point

[10] For a sufficiently many times differentiable function $f(x, y)$ its Taylor's expansion about $(-1, 2)$ is a series in powers of
[A] $x + 1$ and $y - 2$ [B] $x - 1$ and $y + 2$
[C] $x - 1$ and $y - 2$ [D] $x + 1$ and $y + 2$

Q: 2. Answer TEN of the following.

[1] Prove that every convergent sequence is bounded.

[2] Define : (i) Sequence divergent to $-\infty$ (ii)Finitely Oscillating sequence

[3] Show that the sequence $\{S_n\}$ where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ cannot converge.

[4] If $\sum_{n=1}^{\infty} u_n = u$ and $\sum_{n=1}^{\infty} v_n = v$ then prove that $\sum_{n=1}^{\infty}(u_n - v_n) = u - v$

[5] Test $\sum_{n=1}^{\infty} \frac{2n+1}{n}$ for convergence.

[6] Show that the series $1 + \frac{1}{2!} + \frac{1}{3!} + \ldots$ is convergent.

[7] Show that the following function is discontinuous at $(2, 3)$

$$f(x,y) = \begin{cases} 2x + 3y^3 \text{ ; when } (x,y) \neq (2,3) \\ 0 \quad \text{ ; when } (x,y) = (2,3) \end{cases}$$

[8] Evaluate : $\lim_{(x,y)\to(3,1)} \frac{\tan^{-1}(xy-3)}{\tan^{-1}(2xy-6)}$

[9] Evaluate : $\lim_{(x,y)\to(1,1)} \frac{4^{(x-y)} - 1}{x - y}$

[10] Can a function $f(x, y) = x^2 + 5xy + y^2$ have an extreme value at $(1, 1)$? Why?

[11] State the necessary conditions for a function $z = f(x, y)$ to attain extreme values at a point (a, b)

[12] Show that $(y - x)^4 + (x - 2)^4$ has a minimum at $(2, 2)$.

Q: 3 [A] State and prove the Bolzano-Weierstarss theorem for sequence 5

[B] State and prove the Cauchy's criteria for the convergence of a sequence 5

OR

Q: 3 [A] If the sequences $\{a_n\}$ and $\{b_n\}$ are such that
(i) $a_n \leqslant b_n$; $\forall\, n$ and
(ii) $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ 5
then prove that $a \leqslant b$.

[B] If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, 5
then prove that $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{a}{b}$, where $b \neq 0$

Q: 4 [A] State and prove the comparision test of second type. 5

[B] Show that a positive term series converges *iff* the sequence of its partial sums is bounded above. 5

OR

Q: 4 [A] State and prove *D' Alembert's* ratio test. 5

[B] Test for convergence the series whose n^{th} term is $(n^3+1)^{\frac{1}{3}} - n$. 5

Q: 5. If $z = f(x, y)$ is a function of independent variables x, y and if x, y are changed to new independent variables u, v by the substitution $x = \phi(u, v)$; $y = \psi(u, v)$, then express the derivatives of z with respect to x, y in terms of u, v and the derivatives of z with respect to u, v. 10

OR

Q: 5 [A] For the following function show that the repeated limits exist but the double limit does not when $(x, y) \to (0, 0)$

$f(x, y) = \frac{x^2y^2}{x^4+y^4-x^2y^2}$ 5

[B] Define Limit of a function and by using the definition of limit prove that :

$\lim_{(x,y)\to(1,2)} 3xy = 6$ 5

Q: 6 [A] State and prove Taylor's theorem 5

[B] A rectangular box open at the top is to have a volume of $32m^3$. Find the dimensions of box so that the total surface area is minimum. 5

OR

Q: 6 [A] Find the expansion of $\sin x \sin y$ about $(0, 0)$ upto and including and the terms of fourth degree in (x, y). Also compare the result with that you get by multiplying the series for $\sin x$ and $\sin y$. 5

[B] Define Extreme Value and find the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$ 5

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