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[56/A16] SEAT No.________

SARDAR PATEL UNIVERSITY

B.Sc. SEM- V EXAMINATION

22^{th} October 2018 , Monday

10.00 a.m to 1:00 p.m

Sub.: Mathematics (US05CMTH01)

(Real Analysis-I)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) In $(0, \pi/2)$, the function $C(x)$ is
(a) strictly decreasing (b) strictly increasing (c) stationary (d) none

(2) The supremum of the set $\{1 + (-1)^n : n \in N\}$ is
(a) 0 (b) 1 (c) 2 (d) not exists

(3) The set $\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, ...\}$ is...
(a) open and closed set (b) open set (c) closed set (d) neither open nor closed

(4) The field which does not have the least upper bound property is ...
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ (d) $\mathbb{R}$

(5) The derived set of a set is....
(a) open and closed set (b) not open set (c) closed set (d) open but not closed

(6) Every uniformly continuous function is.....
(a) continuous (b) not continuous (c) unbounded (d) none

(7) A function f is said to have a dicontinuity of the first kind from right at $x = c$ if...
(a) $\lim_{x \to c^-} f(x) \neq f(c)$ (b) $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$ exists but not equal
(c) $\lim_{x \to c^+} f(x) \neq f(c)$ (d) None

(8) The Set $\bigcup(\frac{1}{n}, 1 - \frac{1}{n})$ is
(a) $(0, 1)$ (b) $\{0, 1\}$ (c) $[0, 1]$ (d) $[0, 1)$

(9) The continuous function on closed interval is.....
(a) not bounded (b) open set (c) bounded (d) none

(10) If $f'(c) < 0$, then function f is....... at c.
(a) not derivable (b) increasing (c) descreasing (d) none of these.

Q.2 Attempt any **ten** in short: [20]

(1) Is every continuous function derivable? Justify your answer.

(2) Find the supremum and the infimum of a set $\{1 + \frac{(-1)^n}{n} : n \in N\}$.

(3) Prove that supremum of a set S of numbers, if it exists, is unique. [PTO]

(4) Let $m = inf(S)$, where S is a bounded set. Show that $m \in$ closure of S .

(P.T.O.)

(5) Prove that $L(ab) = L(a) + L(b)$.

(6) Prove that the superset of a neighbourhood(nbd) of a point x is also a nbd of x.

(7) If f is derivable at point c then, show that function $1/f$ derivable at point c, where $f(c) \neq 0$.

(8) In usual notations, prove that $\lim_{x \to a}(f - g)(x) = l - m$.

(9) Prove that limit of a function is unique, if exists.

(10) Is $(S \bigcap T)' = S' \bigcap T'$? Justify your answer.

(11) Prove that $a^n = a.a.a......a$ (n times).

(12) Show that the supremum of a bounded non-empty set S, when not a member of S, is a limit point of S.

Q.3(a) Prove that the set of rational numbers is not order complete. [6]

(b) If a be a positive real number and b any real number then there exists a positive integer n such that $na > b$. [4]

OR

Q.3(c) In usual notations, prove that $E(x) = e^x$ for all $x \in R$. [6]

(d) Show that there is no rational number whose square is 5. [4]

Q.4(a) State and Prove Bolzano-Weierstrass theorem for a set. [5]

(b) Show that every open set is a union of open intervals. [5]

OR

Q.4(c) Prove that there exists a positive number π such that $C(\pi/2) = 0$ and $C(x) > 0$ for $0 \leq x < \pi/2$. [5]

(d) Show that a set is closed iff its complement is open. [5]

Q.5(a) In usual notations, prove that $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{l}{m}$, provided $m \neq 0$. [5]

(b) If a function f is continuous on $[a, b]$ and $f(a) \neq f(b)$, then Prove that it assume every value between $f(a)$ and $f(b)$. [5]

OR

Q.5(c) Show that a function $f : [a, b] \to \mathbf{R}$ is continuous at point c of $[a, b]$ iff $\lim_{n \to \infty} c_n = c \Rightarrow \lim_{n \to \infty} f(c_n) = f(c)$. [5]

(d) Examine the function for continuity at $x = 0, 1$ and 2. Also discuss the kind of discontinuity where $f(x)$ defined on $\mathbf{R}$ by [5]

$$f(x) = \begin{cases} -x^2, & \text{if } x \leq 0 \\ 5x - 4, & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}.$$

Q.6(a) Show that a function which is continuous on a closed interval is uniformly continuous on that interval. [5]

(b) Show that $log(1 + x)$ lies between $\frac{x}{1+x}$ and x for all $x > 0$. [5]

OR

Q.6(c) State and prove Darboux's theorem for derivable function. [5]

(d) Define uniform continuity. Prove that the function x^2 is uniformly continuous on $[-1, 1]$. [5]

—x—

(3)