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Seat No : ________

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - V) EXAMINATION
Wednesday , 20^{th} Nov.,2019
MATHEMATICS : US05CMTH05
(NUMBER THEORY)

Time : 10:00 a.m. to 01:00 p.m. | Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) If b/a then $(a, b) =$ .... $\forall\ a, b \in \mathbb{Z}$.

(a) a (b) $|a|$ (c) $|b|$ (d) b

(2) $(a, 0) =$ .... , $\forall\ a \in \mathbb{Z}$

(a) $-a$ (b) $|a|$ (c) a (d) 0

(3) If n is odd integer then $3^n + 1$ is divisible by ....

(a) 5 (b) 3 (c) 4 (d) 6

(4) P(10) = ...

(a) 100 (b) 80 (c) 18 (d) 10

(5) S(60) = ...

(a) 61 (b) 60 (c) 12 (d) 168

(6) .... is Fermat's number .

(a) 4 (b) 6 (c) 17 (d) 15

(7) $ax + by = c$ has integer solution if and only if ....

(a) (a , b) = a (b) (a , b) = b (c) $(a, b)/c$ (d) $c/(a, b)$

(8) Prove that every number containing more than three digits can be divided by 8 iff the number formed by .... digits can be divided by 8.

(a) last two (b) last three (c) first two (d) first three

(9) $\phi(m) \leq$ .... , $\forall\ m > 1$.

(a) m-1 (b) m (c) m+1 (d) m - 2

(10) If a is any integer and p is prime then $a^p \equiv$ ....$(mod\ p)$.
(a) 1 (b) p (c) a (d) a^{p-1}

(PTO)

Que.2 Answer the following (Any Ten) 20

(1) If $\exists\, x, y \in \mathbb{Z}$ such that $xa + yb = 1$ then prove that $(a, b) = 1$.

(2) Prove that (a , b) = d iff the following conditions are satisfied :
(i) d/a , d/b (ii) whenever c/a and c/b then c/d .

(3) Prove that $[a, b, c] = [[a, b], c]$.

(4) If $a = {p_1}^{a_1}{p_2}^{a_2}{p_3}^{a_3}...{p_k}^{a_k}$, where all p_i are primes and $a_i > 0$, then prove that $T(a) = \prod_{i=1}^{k}(a_i + 1)$.

(5) Prove that $\sum_{i=1}^{n} {u_i}^2 = u_n u_{n+1}$.

(6) If $m = qn + r$ then prove that $(u_m, u_n) = (u_n, u_r)$.

(7) If $a_1 \equiv b_1 (mod\, n)$ and $a_2 \equiv b_2 (mod\, n)$,then prove that $a_1 a_2 \equiv b_1 b_2 (mod\, n)$.

(8) If $ca \equiv cb(mod\, n)$ and $(c, n) = 1$ then prove that $a \equiv b(mod\, n)$.

(9) Is 765432 divided by 4 or 7 ? Verify it .

(10) Prove that a set of k integers $a_1, a_2, a_3 \ldots, a_k$ is a complete residue system modulo m iff
(i) $k = m$ (ii) $a_i \not\equiv a_j (mod\, m)$, $\forall\, i \neq j$.

(11) State and prove Fermat's theorem.

(12) Find $\phi(243) + \phi(81) + \phi(27) + \phi(9) + \phi(3)$.

Que.3 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + ... + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 5

(b) State and prove unique factorization theorem for positive integers. 5

OR

Que.3 (c) Prove that $[a, b](a, b) = ab$, $\forall\ ab > 0$. 5

(d) If m is composite integer and $n_m = 1111\ldots$(m times) then prove that n_m is also composite number . 5

Que.4 (a) Prove that odd prime factor of M_p $(p > 2)$ has the form $2pt + 1$, for some integer t. 5

(b) Prove that $S(a) < a\sqrt{a}$, $\forall\ a > 2$. 5

OR

Que.4 (c) Prove that odd prime factor of $a^{2^n} + 1 (a > 1)$ is of the form $2^{n+1}t + 1$,for some integer t . 6

(d) Prove that $u_{n+1}^2 = u_n^2 + 3u_{n-1}^2 + 2[u_{n-2}^2 + u_{n-3}^2 + \cdots + u_1]$. 4

Que.5 (a) Prove that the integer solution of $x^2 + 2y^2 = z^2$, (x , y) = 1 can be expressed as $x = \pm(a^2 - 2b^2)$, $y = 2ab$, $z = a^2 + 2b^2$. 6

(b) Find positive integer solution of $y - \frac{x + 3y}{x + 2} = 1$. 4

OR

Que.5 (c) Prove that a general integer solution of $x^2 + y^2 + z^2 = w^2$, $(x, y, z, w) = 1$ is given by $x = (a^2 - b^2 + c^2 - d^2)$, $y = 2ab - 2cd$, $z = 2ad + 2bc$, $w = a^2 + b^2 + c^2 + d^2$. 6

(d) Find general solution of $50x + 45y + 60z = 10$. 4

Que.6 (a) State and prove Chinese remainder theorem. 5

(b) Solve the system of congruences $x \equiv 2(mod\ 3)$; $x \equiv 3(mod\ 5)$; $x \equiv 2(mod\ 7)$. 5

OR

Que.6 (c) Prove that Euler's function is multiplicative function. 5

(d) Prove that m is prime iff $\phi(m) + S(m) = mT(m)$. 5

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